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ABSTRACT

Mathematics and Living Things (MALT) is designed for grade eight to enrich and supplement the usual courses of instruction. MALT utilizes exercises in biological science to derive data through which mathematical concepts and principles may be introduced and expanded. The Teacher's Commentary includes suggestions for instruction, a list of needed equipment and supplies, a list of things to do to have materials ready for each chapter, background information and a section by section discussion of each chapter, and answers to student exercises. (RH)

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**SCHOOL
MATHEMATICS
STUDY GROUP**

MATHEMATICS AND LIVING THINGS

Teachers' Commentary

(Revised Edition)

U.S. DEPARTMENT OF HEALTH
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MATHEMATICS AND LIVING THINGS

Teachers' Commentary

(Revised Edition)

The following is a list of all those who participated in the preparation of this volume:

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PREFACE

Mathematics and Living Things represents an extension of the series of units designated Mathematics Through Science which the School Mathematics Study Group prepared in summer 1963 and revised, on the basis of teacher evaluations from classroom use in the school year 1963-1964, during a summer period of 1964. Mathematics and Living Things is designed for grade eight to enrich and supplement the usual courses of instruction. MALT utilizes exercises in biological science to derive data through which mathematical concepts and principles may be introduced and expanded.

In addition to the writing team of summers 1964 and 1965, SMSG wishes to acknowledge the assistance extended the MALT writers by Ronald A. Kroman, Assistant Professor of Biology, California State College at Long Beach, and John M. Huffman, San Diego County Department of Education. In particular, John Huffman read the final manuscript copy of the eight chapters in the preliminary version.

Mr. Frank Lindsay of California Department of Education served as chairman of the writing groups and coordinated their use and the evaluation of the preliminary text.

SMSG also wants to thank those classes and teachers which used the preliminary version and whose discerning comments furnished the necessary information to rewrite MALT. Selected teachers in California, Delaware, Texas and Washington tried the preliminary edition. They annotated their texts and commentaries and sent these to SMSG to aid the writers of the revised edition. Several teachers were exceptionally helpful in furnishing very critical comments, test items, samples of student work and editorial remarks: Conrad Sanborn and Maxine Williams of Fresno, California; Clyde Garwood of Garden Grove; and Betty Beaumont of San Antonio, Texas.

This excursion into biological science was initiated at the suggestion of an ad hoc committee of biological scientists assembled in March, 1964, consisting of Charles Brokaw (California Institute of Technology), Allan H. Brown (University of Pennsylvania), Hiden T. Cox (California State College at Long Beach), Ralph W. Gerard (University of California, Irvine), J. Lee Kavanau (University of California, Berkeley). The writing team has drawn from their many suggestions a few thoughts appropriate for units in eighth grade mathematics.

It kept in mind that junior high school mathematics teachers do not have laboratory facilities in their classrooms and may not have recent acquaintance with modern biology.

The attempt of the writers of Mathematics and Living Things has been to introduce through simple activities in biology not only basic processes which underlie the living world but also to teach a logical structure of mathematics that arises naturally from the data students accumulate. This may prove a welcome change to teachers and students who have hitherto only experienced conventional sequence of topics. Effort has been made to identify basic elements of the structure of mathematics, to acquaint the student with a more precise mathematical language, and to present a grade placement of concepts appropriate to his maturity. The writers sincerely hope that the following chapters will be a rewarding and exciting experience for the teacher as well as students.

The writers who prepared the Mathematics and Living Things series offer the following informal suggestions. Each concept should be covered to the degree that the student's mathematical background justifies. For example, some students will be familiar with ideas of accuracy, precision and greatest possible error. For these, those sections may be employed as a brief review. Other topics will require careful explanation by the teacher. Sometimes an activity may not need an entire class period to complete. Then the regular textbook can be turned to for additional problem materials. In short, do not bog down. Help the class move along. Skim when desirable; enrich when necessary. Fit MALT to the class.

The following notations are offered teachers who may not be familiar with the current philosophy of science teaching:

1. To the extent that mathematical computations and analysis have entered into biological investigations, biology has become quantitative and more precise.
2. Since living things are so variable, too few measurements may mislead students in arriving at generalizations. For greater validity the data of an entire class should be pooled.
3. Activities with living things do not always yield expected results. Yet there is no such thing as failure in science. No scientist "throws out" data because it is not as anticipated. It may lead to a new or more refined hypothesis. Hence the results of any student's (or team's) observations should be treated with respect.

Table A: Equipment and Supplies Listed by Chapter

Chapter and Activity	Equipment and Supplies
Chapter 1, Activity 1	ruler (metric), pencil, paper, graph paper, leaves
Activity 2	container (small paper milk carton, paper cup, Petri dish) Chromotography paper (commercial or white paper towel) Pencil, paper, graph paper, ruler (metric) Food coloring or water soluble ink such as Parker ink Watches or clock with second hand
Chapter 2, Activity 1	Wrapping paper (30 or 36" width) 60 ft. Tape measure calibrated in $\frac{1}{16}$ inches (6-10 ft.) Foot ruler for each student Mending (or marking) tape Pencil, paper and graph paper Manila tagboard
Optional Activity	Bean, pumpkin or watermelon seeds Milk cartons Plant food Labeling tape Medium heavy acetate (outdated X-ray film) Paper towels Soda straws
Chapter 3, Activity 1	Bags, plastic (blanket bags approximately 27" × 36") 1 per team Access to a tree Graduated cylinder (or similar to measure ml. (cc.)) Graph paper (10 × 10 to 1 inch), ruler, string or rubber bands, pencil, paper

Table A: Equipment and Supplies Listed by Chapter (Continued)

Chapter and Activity	Equipment and Supplies
Chapter 4, Activity 1	Watches or clocks with second hand Paper, pencil, graph paper
Activity 2	(Same)
Chapter 5, Activity 1	Syringe, plastic, without needle--1 per team Tubing, plastic, transparent (e.g., plastic tubing used in aquarium or disposable I. V. tubing from hispital), 18 inches per team Ruler, metric (preferably plastic with center groove) 1 per team Yeast--package dry or cake--1 per class Sugar, food coloring, Scotch tape Block (or book) covered with white paper--2 per team Watches or clock with second hand Paper, pencil, graph paper
Chapter 6, Activity 1	Tin, aluminum, 9" (cake or pie)--1 per team Gelatin--2 packages per class Bouillon cubes (preferably beef)--2 per class Saran wrap, 1 package Graph paper, 10 × 10 to the inch--2 per team plus 8 per student Rubber bands or mending tape--1 per team Scissors--several per class Paper, pencils, rulers
Optional Activities	Laundry bluing Household ammonia Table salt Milk cartons Paper towels Cotton string (10-12 ft.) medium diameter Aluminum pie tins Charcoal briquets--1 per team Heavy acetate (outdated X-ray film)--1 per team

Table A: Equipment and Supplies Listed by Chapter (Continued)

Chapter and Activity	Equipment and Supplies
Chapter 6 contd.	
Optional Activities	Graph paper, 10 × 10 to the inch--1 per team
	Pieces of brick, coal or cinders
	Eye dropper (or soda straws)
	Plastic ruler (metric)
Chapter 7, Activity 1	Manila tag or construction paper (for construction of polyhedrons)
	Protractor, ruler and pencil--1 each per student
	Paper, (newspaper and smaller) large supply
Chapter 8, Activity 1	Tape measure (25 to 100 feet)
	Yard or meter sticks
	Paper, pencil, ruler

Table B: Summarized - Equipment and Supplies List

Equipment or supply	Number needed	Chapters where needed in order of appearance in text	Source	Cost (Approximate)
ruler, mm. and inch	1 per student	in all activities	School Supplies	0.10 each
Pencils	1 per student	in all activities		
Paper	-	in all activities		
Paper	4 per student	1, 2, 4, 5		
Paper, graph 10 × 10 to 1 inch	10 per student	1, 6		
Cups or beakers	2 per team	1, 2	½ pint milk cartons from cafeteria	
Chromotography paper	1 strip per student	1	white paper towel from store Commercial paper from supply house	
Food coloring	2 or 3 drops per student	1, 5	Grocery store	0.30 package of 4
Seeds	5 per team	2	Grocery store (black-eyed beans, pumpkin or watermelon)	0.27 or less per package
Plant food	½ tablet per team	2	many stores	0.39 per pkg.
Tins, aluminum 9"	2 per team	2, 6	many stores (esp. variety stores)	0.49 per pkg. of 6
Tape, label or Scotch	-	2, 5, 6	many sources	varies
Meter sticks	1 per team	2, 8	supply house school supply	
Bags, plastic 27" × 36"	1 per team	3	Variety store	0.69 per pkg. of 4
Graduated cylinder	2 or more per class	3	Science supply, hobby shop	0.30 +

Table B: Summarized - Equipment and Supplies List (Continued)

Equipment or supply	Number needed	Chapters where needed in order of appearance in text	Source	Cost (Approximate)
Syringe, plastic	1 per team	5	Hospital or doctor's office (disposable)	-
Tubing, plastic	18" per team	5	Hospital, doctor's office, or pet store	-
Yeast	1 per class	5	Grocery (school cafeteria)	0.10/pkg.
Sugar	Small amount	5	Cafeteria or home	-
Gelatin	1 pkg.	6	Grocery (maybe school cafeteria)	0.25/pkg.
Bouillon cubes	1 pkg.	6	Grocery	0.10
Saran wrap	1 pkg.	6	Grocery	0.30/100 ft.
Scissors	several	6	Usually supplied by school	-
Bluing	1 bottle per class	6	Grocery store	0.35
Table salt	1 lb. per class	6	Grocery store	0.15
Household ammonia	1 small bottle per class	6	Grocery store	0.19
Manila tag	4 sq. ft. per student	7	From school supplies	-
Protractors	1 per student	7	From school supplies	-
Tape measure	1 per room	8	From P.E. dept.	-

Preliminary Preparations

	Chapter	Pre-prepatation suggestions
1.	MEASUREMENT OF LEAVES	The area should be "surveyed" several days in advance and certain types of trees or shrubs suggested to the students.
2.	ADDITION OF MEASUREMENT	The beans _must_ be planted _five_ or _six days_ before measurement is to begin. Containers must be ready at the time of planting. Having the students bring milk cartons to class a week or two in advance is one suggestion.
3.	LEAF SURFACE AREA AND WATER LOSS	Again, the trees or shrubs to be used should be chosen well in advance of the day planned for the activity. If the microscope exercise suggested in this commentary is to be used, a relatively _thick_ leaf is recommended in order that epidermal tissue may be obtained more easily.
4.	MUSCLE FATIGUE	No special advance preparation, except the timing device.
5.	YEAST METABOLISM	If at all possible, this should be tried by the teacher in advance, This will help the teacher to anticipate problems involved in setting up the exercise. Plastic syringes and I. V. tubing should be requested from doctors or nurses _three or four weeks_ before the activity is planned, so there will be time for them to accumulate the necessary quantity.
6.	GROWTH OF MOLD	If students are to bring the aluminum cake tins suggested for this activity, they should be warned a week or two before, so that all will be ready at the proper time.

Chapter	Pre-prepatation suggestions
6. (continued)	The second activity suggested, "Growth of Crystals," requires a variety of containers. The solution can be mixed in the classroom during the class period. This is an exciting activity and will interest most students.
7. SURFACE AREA AND VOLUME RELATIONSHIPS	Since this does not involve an activity that is biological in nature, no advance preparation is necessary except for having on hand the Manila tag-board for building the models.
8. VOLUME OF A TREE	Suitable trees should be selected by the teacher several days in advance.

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Chapter 1

LEAVES AND NATURAL VARIATION:
MEASUREMENT OF LENGTH, METRIC SYSTEM, RATIO, AND GRAPHING

1.1 Introduction

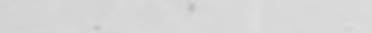
Much has been written and said about the scientific method, scientific approach, and scientific reasoning to the point that one is led to believe that this is a process limited to the exclusive use of practicing scientists. The brief discussion in the introduction is done purposely to point out that everyone, at some time, uses a similar process, though maybe not quite so refined, in solving everyday problems. Students should understand that a single approach (or process) will not "fit" every investigation. It is important to first define the problem (often a question) and then devise a plan to follow. It is just as important to recognize that the first plan may not work and will have to be revised. The following exercises are designed to give students experience in this kind of planning.

1.2 Measure and Units

In Chapter 1 basic concepts of measurement and ratio are introduced. No attempt is made at this point to cover completely the concept of measurement. It is felt that students can best become acquainted with systems of measurement by actually using measuring devices in fairly easy to do exercises. It should be emphasized that measurements can be only as exact as the unit in which the student is measuring. For example, in Exercise 4 the student is instructed to measure to the nearest centimeter and therefore should not attempt to divide centimeters into smaller units.

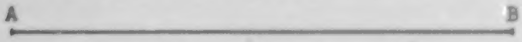
Exercise 1-3

1. Measure the length of this line to the nearest cm .



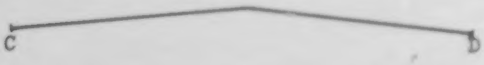
Answer: 5 cm

2. Measure to the nearest cm the distance from A to B.



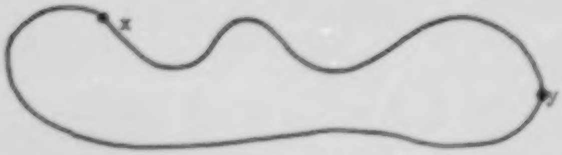
Answer: 7 cm

3. Measure to the nearest cm the shortest distance between C and D.



Answer: 7 cm

4. Measure to the nearest cm the distance between points x and y.



Answer: 7 cm

5. Measure to the nearest millimeter each of the problems above.

Answer: 53 mm, 74 mm,

68 mm, 66 mm

6. How do the cm and the mm measurements of Problem 1 compare in exactness? Which is closer to the exact measurement?

Answer: mm is more exact

the mm measure

7. How do the cm and the mm measurements of Problem 3 compare in exactness?

Answer: mm more exact

Which answer is closest to the "actual" measure?

Answer: mm

Is your answer exact? Is any measure exact?

Answer: no, no

Answer: This is meant as a challenge to the student. "Exactness" is discussed in Section 1-5.

8. Measure the length of 1 inch in millimeters. About how many millimeters are there in 1 inch?

Answer: 25

1.4 Activity - Measurement of Leaves

Answer to Question

Are these expressions mathematical or scientific?

Neither of the two statements about corn would be considered mathematical or scientific because these are not precise criteria. Both knees and elephant eyes vary in height!

Preliminary Preparation

Materials Needed

Metric rulers
Leaves
Graph paper
Data book (see Procedure)

When instructing the student concerning the obtaining of leaves for measurement, the following things should be considered:

Type of leaf

Any type similar to the illustration in Fig. 1-4a will work as long as its outline is relatively even.

It is extremely important that all leaves used by one student or pair of students be from the same or the same kind of tree, bush, or vine (the same species).

It would be more scientific if the teacher could obtain permission for students to get all their leaves from the same species on the school grounds. The advantage of this is to point out the relative validity of data obtained from a large sample in comparison to the small sample which would result from each student bringing his own. If all could use the same, a large chart where students could pool their data would be helpful. Special emphasis should be placed on not raiding neighbor's yards or collecting on the way to school without permission. It is suggested that the teacher have available a supply of leaves or branches for those who forget.

Procedure

This is an activity which could be done either individually or in pairs, with one person measuring, while the other records.

Points to emphasize

Review the importance of the measurement of width at the midpoint of the length measurement, not the widest point of the leaf, since this often is a difficult thing to determine.

The table has been set up with width in the first column and length in the second because the first number of the ordered pair (the domain) has by tradition always been the horizontal distance from the origin and the second number (the range) has traditionally been the vertical distance from the origin. If we had reversed the position of length and width on our graph, then the ratio L/W would not represent a slope of the function (see Section 1.12).

It is suggested that some sort of semi-permanent device be used for recording data. If the texts are not to be reused, data could be recorded directly in the tables provided. If this is not feasible, then students could construct their own tables in a notebook kept especially for this purpose, in which case each table should be appropriately and completely labelled. In some cases, including this activity, the data recorded at this time will be used later in the text to help develop or illustrate other mathematical principles (mean, frequency histogram, sampling).

Exercise 1-5

Measures and Units

1. Pick out the measures and the units in each of the following measurements.

	Answers	
	Measures	Units
(A) 21 feet	21	feet
(B) 7 yards	7	yard
(C) 10 hands	10	hand
(D) 15 centimeters	15	centimeter

2. If a fish tank were filled by emptying a gallon bucket into it 8 times, what would be the volume of the tank? What is the measure? What is the unit?

Answers: 8 gallons, 8, gallon

3. The fish tank of problem 2 is filled by using a quart container rather than a gallon container.

(A) What is the measure?	32
(B) What is the unit?	quart
(C) Does the volume remain the same?	Yes, since 32 qts = 8 gal
(D) Is the measurement the same?	No, in one case the measure is 8 gal., in the other 32 qts.

4. Change each of the following measurements to an equal measurement having a different measure and unit.

(A) 2 hours
(B) 2 minutes
(C) 3 square feet
(D) 1 square yard

There are several possible answers to each of these.

5. Change the following leaf measurements to centimeters.

(A) 25 mm = 2.5 cm
(B) 70 mm = 7.0 cm
(C) 4 mm = .4 cm
(D) 7 cm, 3 mm = 7.3 cm

1.6 Measurement of Length: General

The millimeter is the most suitable metric unit for measuring this size of leaf. The point to stress here is that, although other units could be used under other circumstances, this is the most useful one for this exercise--in our time, in our society, and with present practices.

Exercise 1-7

Ideas of Accuracy

1. Suggest commonly used units for the following measurements.

	Answers
(A) the altitude of an airplane;	Feet or Miles
(B) the length of a car;	Inches or Feet
(C) the depth of the ocean;	Feet or Fathoms
(D) the length of an arm;	Inches
(E) the height of a tree;	Feet
(F) the height of a truck	Inches

Other responses might be equally acceptable. Discuss them.

2. What unit of measure would be commonly used when measuring the width of a window for drape rods?

Inches

3. What unit of measure would be commonly used when measuring the width of a window to fit glass?

Quarter inches
(considerable tolerance -
covered with putty)

4. What statement concerning choice of units of measurement is demonstrated by your answers to the questions above?

Answer: The purpose of the measurement will usually suggest the most suitable unit.

1.8 Natural Variation

The table of leaf measurements should show very nearly a "normal curve" if their leaves were chosen as instructed. In any case, a variation will appear, which illustrates the point. The same type of distribution should appear in the average class. An interesting discussion can arise, however, if the class is not average. Is there a preponderant number of large people? Some factors that might account for this might be age, or race (Scandinavian, for instance).

The students should be able to suggest a number of examples of natural variation--and literally almost "anything goes" !

Viruses and minute insects such as gnats and mosquitoes are other examples of "successful, even though not big."

Insects are the most diversified of all animals. There are at least 600,000 different species known--and more are being discovered constantly. Some of their adaptations are truly remarkable. The "leaf mantis" of South Eastern Asia is perhaps one of the most startling. Equally dramatic are the walking stick and the leaf moth. An excellent reference for this chapter is Nature's Ways, by Roy Chapman Andrews, Crown Publishers, New York. Fine material is also available in the Life Magazine Nature Library.

The discussion of internal variation is a brief introduction to the thread of metabolism which runs through the rest of the chapters.

1.9 Ratio

This portion should be just a quick review of "ratio." Stress the importance of the order in a ratio relationship. In the case of all definitions be sure the student forms a good habit and refers to the entire definition. $d \neq 0$ is necessary in the definition. For example, the ratio $\frac{2}{0}$ has no meaning.

The ratio of the number "d" to the number "c" would be $\frac{d}{c}$, $c \neq 0$. ($\neq$ is the symbol for "not equal to")

You may need to review just why a number divided by zero has no meaning. For example, if $\frac{6}{2} = 3$, then $6 = 2 \cdot 3$. If $\frac{6}{0}$ = something, then 6 = something $\cdot\ 0$. We know that 0 times any number $= 0$, therefore $\frac{6}{0}$

cannot equal anything and certainly not 0 (as $0 \cdot 0 \neq 6$)

An analogous situation occurs with $\frac{0}{0}$. If $\frac{0}{0}$ = something, then $0 \cdot 0$ = something. Now we have too many possibilities because $0 \cdot$ any number equals 0.

Exercise 1-9

COMPARING PAIRS OF NUMBERS

	The Numbers		Add	Subtract (2nd from 1st)	Multiply	Divide (1st by 2nd) Record answer in simplified form
1.	5	2	7	3	10	2.5
2.	90	36	126	54	3240	2.5
3.	65	26	91	39	1690	2.5
4.	102	34	136	68	3468	3.00
5.	27	9	36	18	243	3.00
6.	39	13	52	26	507	3.00
7.	51	17	68	34	867	3.00
8.	72	16	88	56	1152	4.5
9.	162	36	198	126	5832	4.5
10.	9	2	11	7	18	4.5

1.10 Ratio: Length to Width

If we are comparing the measure of the length to the measure of the width, the ratio would be $\frac{l}{w}$, with length as the numerator.

This demonstration is a good exercise in approximation. The length and width are both hard to determine on a leaf and the measure, at best, is to the nearest mm.

When we compare or "order" two numbers such as $\frac{100}{67}$ and $\frac{131}{88}$ the process suggested in the text is but one of several ways. However, it is the most common. You could show that $\frac{100}{67} = 1.492537313\ldots$ but the ratio is: $\frac{100}{67} = \frac{1.492537313\ldots}{1}$. The denominator does not disappear; it just is usually not shown just as 2 is seldom written $\frac{2}{1}$.

Another common way of ordering rational numbers is to change both to equivalent fractions with common denominator. Thus $\frac{100}{67}$ and $\frac{131}{88}$ have a common denominator of $67 \cdot 88$ or 5896. The equivalent fractions then would be $\frac{100}{67} = \frac{8800}{5896}$ and $\frac{131}{88} = \frac{8777}{5896}$. Thus $\frac{8800}{5896} > \frac{8777}{5896}$ and $\frac{100}{67} > \frac{131}{88}$.

When changing from a fractional number to an equivalent decimal, the unit or units of comparison are the units used in the decimal system, tens, ones, tenths, hundredths, etc.

"Rounding off" might need to be reviewed.

Rounding off is useful in estimating results. For instance, suppose we have to find the product: 1.34×3.56. This would be approximately $1 \times 4 = 4$, or, if we wanted a little closer estimate, we could compute : $1.3 \times 3.6 = 4.7$ approximately.

Rounding off is also useful when we are considering approximations in percents. For instance, if it turned out to be true that about 2 out of 7 families have dogs, it would be foolish to carry this out to many decimal places in order to get an answer in percent. We would usually just use two places and say that about 29 percent of all families have dogs, or we could round this still further and refer to 30 percent.

A special problem arises when the number to be rounded occurs exactly half-way between the two approximating numbers. For instance, how does one round 3.1215 to three decimal places? The two approximating numbers are 3.121 and 3.122 and one is just as accurate an approximation as the other. Often it makes little difference which decimal is used. However, if for several numbers of a sum one always rounded to the lower figure, the answer would probably be too small. For this reason, we shall agree to choose the decimal whose last digit is even. Thus in the above case, we could choose 3.122 since its last digit is even; this number is larger than the given number. But if the given number were 3.1425 we would choose 3.142 as the approximating number since the given number lies between 3.142 and 3.143 and it is the decimal 3.142 whose last digit is even; here we have chosen the smaller of the two approximating numbers.

1.11 Average Ratio

Leaves picked from the same tree or bush will usually have approximately the same ratio of $\frac{\ell}{w}$. However, the ratio of the very small young leaf and the very mature large leaf quite often differs from the mean. This, of course, is to be expected and would show up nicely on a frequency distribution.

The students should all save their data tables and graphs. The data on average ratios could be used later after frequency distribution has been introduced. You may even decide to make a class frequency distribution based upon each individual ratio. A sample of 600-800 leaves could give a nicely formed normal curve.

1.12 Graph

A good commercial graph paper available in the school store or supply would be an advantage to the construction of many graphs in this unit. A choice of papers would be even better. 10 × 10 to the $\frac{1}{2}$ inch, 10 × 10 to the inch, centimeter squares, and $\frac{1}{4}$ inch squares are all handy types of graph paper to have.

The concept of ordered pairs is very informally introduced here. Ratio, with its emphasis on the order, leads into ordered pairs. However, we run into a problem here in this demonstration. If we graph ℓ/w as (ℓ, w) then the length is the horizontal distance from the origin, and the width is the vertical distance from the origin. This would be all right except for the conflict with the "slope" of the line. Line slope is $\frac{\Delta y}{\Delta x}$ (amount of "y" change over the amount of "x" change.) A ratio of $\frac{\ell}{w} = \frac{72}{40} = 1.8$ means that length increases 1.8 units for every 1 unit width increase. On a graph the slope of 1.8 (line A) would be "steeper" than a slope of 1 (line B). See Figure 1-12a. But if graphed as "ℓ" for abscissa (first term - horizontal distance) and w for ordinate, then the graph would be a line (line C) which appears as a slope less than 1. This is the reason students were instructed to record widths in the left column and lengths in the right column of their data tables. Ordered pairs of the form (w, ℓ) give a linear graph whose slope is the same as their average ratio $\frac{\ell}{w}$. The writers felt that this consistency was important.

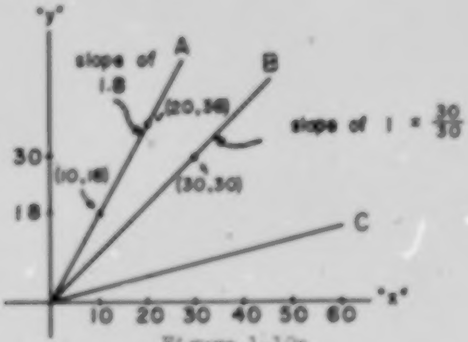


Figure 1-12a

Exercise 1-12

Problems Using Ratio

1. A tree in front of a fraternity house on the campus of Stanford University gave an average ratio of $\frac{\ell}{w}$ of leaves as 1.4 for the leaves measured. If a leaf had a width of 50 mm what would be its length?

70 mm

2. From the same tree as in Problem 1

   a. If a leaf were 65 mm in width, what would be the length? 91 mm

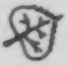
   b. A width of 20 mm would give a length of ____. 28 mm

3. a. If the length of a leaf from the same tree was 47 mm, what would be the expected width? 33 mm

   b. From a length of 65 mm, one would expect a width of _______. 46 mm

4. a. Describe a leaf which had a $\frac{l}{w}$ ratio of .65.

   b. Draw what you think such a leaf would resemble. Short and wide



5. If a leaf had a $\frac{w}{l}$ of .65 and a width of 35 mm, what would be the expected length? 54 mm

6. With a $\frac{w}{l}$ ratio of .72 and a length of 67 mm, what would be the expected width? 48 mm

7. a. Given a $\frac{l}{w}$ of $\frac{1.4}{1.0}$. Find the expected length of a leaf which has a width measure of 1 mm. 1.4 mm

8. a. Given the same ratio of $\frac{1.4}{1.0} = (\frac{l}{w})$ what would be the mathematical expectation of the length measure if the width measure were 35 meters? 49 meters

   b. Would such a leaf be found on Earth? not likely

J.F.F. (Just For Fun)

A few of these are inserted "just for fun." Don't tell the answers. Let those who figure them out have all of the pleasure.

If a brick weighs 9 pounds and a half, what is the weight of a brick and a half?

Answer: 27 lbs.

One solution:

Let x = weight of a brick, then $x = 9 \text{ lbs.} + \frac{x}{2}$ is an equation.

Solve for x $\quad 2x = 2 \cdot 9 + 2(\frac{x}{2})$ multiply terms by 2

$2x = 18 + x$ another name for 1

$x = 18$

Therefore, if a brick weighs 18 lbs. then $1\frac{1}{2}$ bricks weigh 27 lbs.

Background Information

A Little Bit About Leaves

Leaves in general perform for the total plant the function of food manufacture, called photosynthesis (building with light). Several characteristics which contribute to the carrying out of this function are immediately apparent.

Most leaves contain chlorophyll, a green pigment whose purpose is to utilize light energy (it need not necessarily be sunlight) to produce--through a series of chemical reactions--food for the plant, and incidentally, for all animals as well. Some leaves are variegated, having either light areas containing little chlorophyll, or dark red or brown areas consisting of darker pigments covering the chlorophyll, but not hindering its operation. There are other pigments as well in most leaves--chlorophyll b, xanthophyll, and carotene for example, each serving a supporting role in photosynthesis.

Leaves, except for a few with special adaptations, some of which are described below, are almost universally flat.

Food manufacture requires, in addition to chlorophyll and light, the raw materials water from the soil and carbon dioxide from the air. The water is carried through specialized tubes from the roots via the stem petioles, if present, to the leaf blades where the tubes can be seen clustered together in veins. The veins spread over a leaf in a pattern, the type of pattern depending on the type of plant. In general, these plants can be divided into two categories: those with parallel-veined leaves as illustrated by the lily family or corn, and those with netted-veined leaves, as shown in the text.

Another important function of leaves is the control of water loss. The tiny openings in leaf surfaces are called stomates--or stomata, to use the Latin term. (These are invisible except with a microscope.) Stomates open and close by reacting to several factors in the cells and the atmosphere, such as water pressure, light, and temperature. Most leaves also have added protection from water loss, for example, a waxy or hairy coating, or modification into spines.

The stomates also control the exchange of gases between the cells of the leaf and the environment. Carbon dioxide is essential for photosynthesis and oxygen is released as a by-product, a process occurring only in the presence of light. At the same time, oxygen is needed for respiration (release of energy) and carbon dioxide is the by-product. This process is going on constantly, whether light is present or not.

Some unusual adaptations among leaves:

Sweet Pea tendrils are modified leaves used for clinging.

Pitcher Plant and Venus Fly Trap are both modified to capture insects. They grow in bogs where essential nitrogen is scarce and apparently obtain the nitrogen from the animal protein.

Water Lilies have broad flat leaves adapted for floating.

Cactus "leaves" are modified into spines for water conservation. The broad structures are stems.

Conifers (cone-bearing trees) have needles or scale-like leaves.

Some amazing facts:

It has been estimated that annually 200 billion tons of carbon from carbon dioxide is converted into starch by plants! The process itself (not the quantity) is equivalent to a car being able to convert its own exhaust gases into fuel.

The process of photosynthesis has often been quoted as the single most important chemical reaction in the world, since the entire food supply and nearly all the total fuel supply (wood, coal, oil) is dependent upon it. The food of all living things is ultimately traceable back to plants, even though the so-called "food chain" may include many animals.

Example: grain mouse snake hawk.

Chlorophyll is organized into tiny chloroplasts, averaging between 5 and 10μ long. (A micron (μ) is $\frac{1}{1000}$ of a mm.) "100 chloroplasts placed end to end would form a link sausage that would just stretch across the diameter of the period at the end of this sentence," states J. Van Overbeek in The Lore of Living Plants, McGraw Hill, 1964.

1.13 Activity 2 - Paper Chromatography

No attempt is made to use scientific terminology pertaining to chromatography. Although this technique is commonly used as an aid in chemical analysis, it is used here only as a means of collecting data that can be used to further develop mathematical concepts. It should be pointed out that the materials and techniques used in the experiment only demonstrate the method and will not result in data that can be used for precision analysis. It has been found that easily obtainable materials give results that are satisfactory. The chromatography papers used in trial runs were strips cut from white absorbent paper towels (either roll or folded). The water container was made by cutting the bottom from a paper milk carton (paper cup will do) leaving sides approximately one-half inch high. The substances for color separation could be such things as water soluble inks, food coloring, cloth dyes, etc. (Parker fountain pen ink or blue food coloring is good.) More uniform data will result if students are cautioned not to let the over-hanging paper strip touch the container or desk edge. Also the rate of travel by the water is affected by evaporation so care should be taken to set up apparatus in areas as free from air currents or drafts as possible. In several trials using white absorbent paper towel and blue food coloring the data resulted in a graph similar to the two shown in Figure 1-13 below.

In some cases where regular chromatography paper is available, it is recommended that it be used. (Try the local chemistry teacher.)

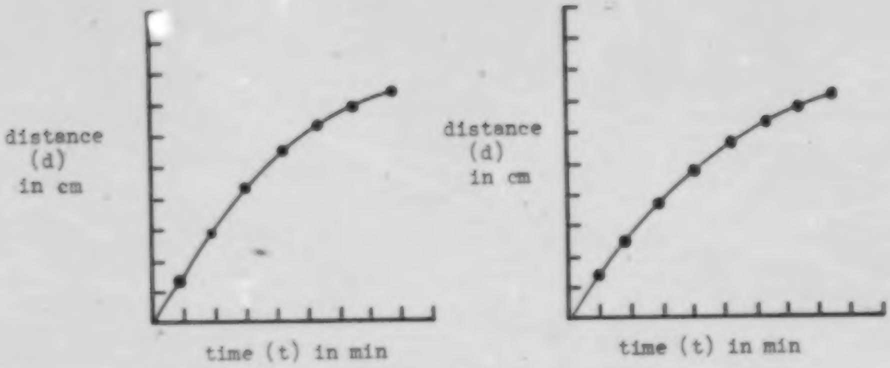


Figure 1-13

Materials needed for each student:

1. Substance to be tested. (Ink or food coloring)
2. Shallow container for water (solvent). Bottom of a small paper milk container or paper cup.
3. Paper (chromatography or white paper towel).
4. Paper clip
5. Water
6. Pointed object such as toothpick
7. Ruler (preferably metric)
8. Clock or watch
9. Graph paper

Procedure

Some students may question the marks placed on the paper strip. If they think that the pencil mark is hindering the movement of solvent then have them try the same size strip but change the pencil mark to a "dot" along one side.

The amount of test substance placed on the chromatography paper should be limited to a small dot if possible. Ten minutes should be adequate time to give results for both determining Rf values and graphing.

Chromatography Data

Table II

	Distance Traveled	Ratio Front (Rf)
distance traveled by water	12.5 cm	1.00
distance traveled by brown	11.7 cm	.94
distance traveled by blue	9.90 cm	.79
distance traveled by red	5.50 cm	.44

Table 1-13b

1.14 Rf Value - Ratio

The data can be used to determine Rf values by finding the ratio of distances traveled.

$$\text{Rf} = \frac{\text{Distance traveled by color}}{\text{Distance traveled by water (solvent)}}$$

It is important to point out that standard tables of Rf values include information on type of paper, solvent used, temperature, and time. In other words, a given substance could have a different set of Rf values for each variable. The Rf values given as examples in the student text do not state the conditions.

Exercise 1-14

Rf Values

1. Interpret the following Rf values with respect to distance traveled by water and colors.

	Problem	Answer
a.	Rf = 0.1	Color travels 0.1 × water distance
b.	Rf = 0.5	Color travels $\frac{1}{2}$ × water "
c.	Rf = 0.8	Color travels .8 × water "
d.	Rf = 0.25	Color travels .25 × water "
e.	Rf = 0.33	Color travels .33 × water "
f.	Rf = 1.0	Water and color travel same "

2. a. A chromatogram produced the following data. Identify as A, B, C, etc. the compounds in the mixture. (Data in student text)

distance traveled by water = 25 cm
distance traveled by Color I = 25 cm
distance traveled by Color II = 10 cm
distance traveled by Color III = 4 cm

Answer: Compounds E, B, H.

b. (Data in student text)

distance traveled by water = 18 cm
distance traveled by Color I = 12 cm
distance traveled by Color II = 16.6 cm
distance traveled by Color III = 5.8 cm

Answer: Compounds C, A, G.

1.15 Graphing Chromotography Data

Students are instructed to refer to Section 1.12 if a review of graph making is necessary.

Questions are asked about ease of interpretation (data table versus graph) and in this case they may "see" an easy interpretation from data table. Point out, however, that this is not always the case. Generally, it is easier to make a general statement of interpretation when reading a graph.

Exercise 1-15

Interpreting Graph Data

1. What is the measure of the distance that the mixture traveled during the first minute?

   Answer: Will vary

2. Find the measure of the distance traveled by the mixture during the second minute.

   Answer: Will vary

3. Did the mixture travel the same distance each minute?

Answer: No

4. How far did the mixture travel during the fifth minute?

Answer: Will vary

5. What is the ratio of the distance traveled during the second minute to distance traveled during the first minute?

Answer: Will vary

6. What is the ratio of the distance traveled during the fifth minute to distance traveled during the first minute?

Answer: Will vary

7. Approximately how many mm did the mixture travel during the last minute?

Answer: Will vary but probably less than 5 mm

8. Express as a ratio the distance traveled during the last minute to the distance traveled the first minute.

Answer: Will vary

9. At what time interval does the "slowing down" process first become really noticeable?

Answer: About the fifth minute

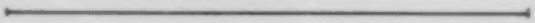
10. Given a paper of infinite length and conditions the same as in your classroom, do you think the process would ever stop completely? Why?

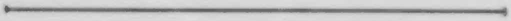
Answer: Yes. As surface area becomes greater, evaporation becomes greater and equilibrium between capillary action and evaporation would exist.

Sample Test Items

Measurement items

1. Measure the length of the following segments to the nearest centimeter.

(a)  (8 cm)

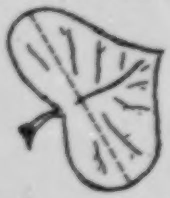
(b)  (7 cm)

2. Measure the segments in the problem above to the nearest millimeter.

   (a) 77 mm

   (b) 74 mm

3. Using the methods of the last few weeks, measure the length and the width of the following leaf to the nearest millimeter.



l = 20 mm

w = 39 mm

4. Name the following quantities using a different unit of measure.

(a) $3\frac{1}{2}$ hours	(210 minutes)
(b) 12 minutes	($\frac{1}{5}$ hr or 720 sec)
(c) 2 yd	(6 ft)
(d) 4 cm	(40 mm)

5. The term that we use to explain the fact that some people of the same age are taller than others, is __________. (natural variation)

6. What would be a logical unit to use in the following measurements?

(a) The height of a telephone pole	(feet)
(b) The width of a desk	(inches)
(c) Depth of your aquarium	(inches)
(d) Length of your car's wheelbase	(inches)

7. What measurement of length can we use and be exact?

   __________ (none)

Ratio Items

8. Express the ratio of 9 to 5 as a fraction. ($\frac{9}{5}$)

9. What is the ratio of 5 to 9 ? ($\frac{5}{9}$)

10. What is the ratio of w to q ? ($\frac{w}{q}$)

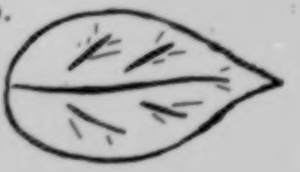
11. Which number is greater, $\frac{5}{9}$ or $\frac{5}{8}$? ($\frac{5}{8}$)

12. $\frac{4}{7}$ of the class at Johnny Fink High School are boys. The total enrollment is 791. How many boys are there in the high school? (452)

13. A leaf had a $\frac{\ell}{w}$ ratio of 1.1. If its width is 1 cm, what is its length in mm ? (11)

14. With a $\frac{\ell}{w}$ ratio of .95, what would be the width of a leaf 15 cm long? Answer in mm. (142.5 mm)

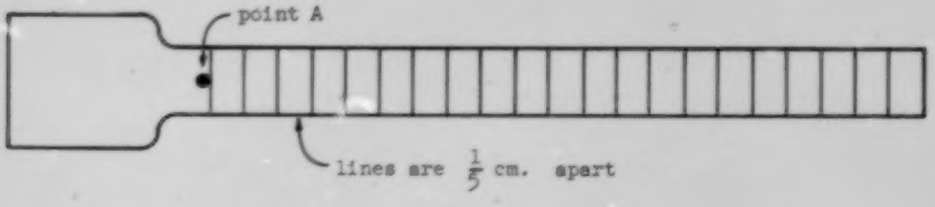
15. Draw a leaf with an approximate $\frac{\ell}{w}$ of 1.9.



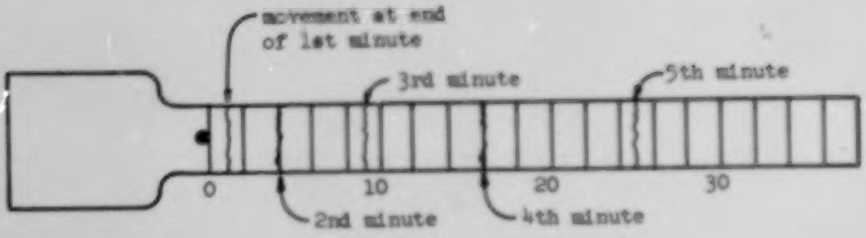
Chromatography Items

Explanation of Experiment 1:

A small drop of brown food coloring was placed on point A of the following paper towel strip:



As in our chromatography experiments, the towel strip was placed in a shallow container of water for the purpose of observing the movement of the brown food coloring down the strip. The following illustrates the results of our experiments:


movement at end of 1st minute
3rd minute
5th minute
0
10
20
30
2nd minute
4th minute

The experiment was stopped at the end of 5 minutes.

Questions on Experiment 1:	Answers
1. How many mm apart are the parallel lines on the paper towel strip?	1. 2 mm
2. Prepare a time in minutes--distance in mm table.	2.
3. How many mm did the food coloring travel during the 4th minute?	3. 7 mm
4. How many mm did the food coloring travel during the 1st minute?	4. 1 mm
5. What is the ratio of the distance traveled during the 3rd minute to the distance traveled during the 1st minute? (decimal notation)	5. 5.0

time in min.	dist. in mm
0	0
1	1
2	4
3	9
4	16
5	25

6. What is the ratio of the distance traveled during the 2nd minute to the distance traveled during the 5th minute? (decimal notation)

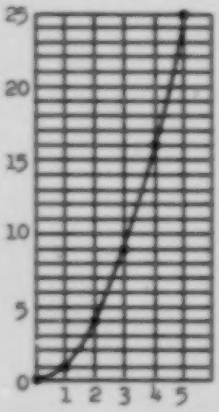
6. .33

7. At what time interval does the color begin to slow down?

7. hasn't yet

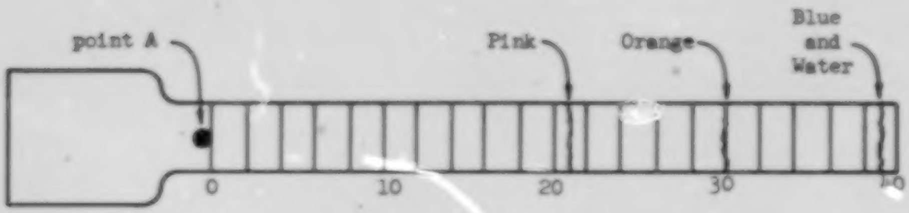
8. Graph the data of the table of question No. 2 using the horizontal axis for units of time and the vertical axis for units of distance traveled. Draw a curve that best fits the points you have plotted.

8.



Experiment 2

One drop each of blue, orange, and pink food coloring are placed on point A. The following paper strip illustrates the distance traveled by each color at the end of 5 minutes.



Questions on Experiment 2:

1. What is the Ratio Front for the color pink? 1. .54

2. What is the Ratio Front for the color blue? 2. 1.00

3. What is the Ratio Front for the color orange? 3. .77

Listed below are some Rf values:

Substance	Rf	Substance	Rf
A	1.2	E	0.7
B	0.5	F	0.2
C	0.4	G	0.9
D	1.0	H	0.3

A chromatogram produced the following data:

Distance traveled by water = 35 cm
Distance traveled by Color I = 14 cm
Distance traveled by Color II = 7 cm
Distance traveled by Color III = 35 cm

4. Color I is the same as Substance ________. 4. C

5. Color II is the same as Substance ________. 5. E

6. Color III is the same as Substance ________. 6. D

Another chromatogram produced:

Distance traveled by water = 16 cm
Distance traveled by Color IV = 19.2 cm
Distance traveled by Color V = 11.2 cm
Distance colored by Color VI = 4.8 cm

7. Color IV is the same as Substance ________. 7. A

8. Color V is the same as Substance ________. 8. E

9. Color VI is the same as Substance ________. 9. H

Chapter 2

NATURAL VARIATION - "US":

ADDITION OF MEASUREMENT AND GREATEST POSSIBLE ERROR

2.1 Introduction

It is the hope of the writers that by using a biological activity and the presentation of a biological problem at the outset, thereby showing the need for understanding of certain mathematical procedures, a higher degree of motivation might be realized.

As in Chapter 1, the mathematical concepts dealt with are not developed to their fullest potential, but are treated only sufficiently to enable the student to solve the problem at hand.

In this chapter measurement is expanded to include addition of measurements and the concept of greatest possible error in measurement, approached through addition, as well as determination of averages.

2.2 Preliminary Preparation

The teacher must plan well ahead for this activity, since some advance work must be done. Students are instructed to read all of Sections 2.2 and 2.3 before starting this activity.

Materials:

1. About 60' of wrapping paper (30 - 36" width).
2. A 6- or 10-foot tape measure, calibrated in $\frac{1}{16}$ -inches.
3. A foot ruler for each student.
4. Masking tape (Scotch is one brand) to hold the wrapping paper secure.

In selecting the location in the room for the measurements, the teacher should point out to the students the difficulties of measuring over chalk rails or any other objects which extend from the wall. The authors found that the wall extending from a corner was an easy place to measure reach by having the students touch the adjacent wall with their finger tips. (Figure 2-4b.) The wrapping paper should be long enough so that no student's reach, or height, would extend beyond the paper.

Some students will have a 6-, 8-, or 10-foot steel tape measure at home that they would be willing to bring for the measuring. It is important that in the foot measure they first measure each individual foot length and then take a single measure with the tape to show their difference in error. The single measure together with its G. P. E. should fall well within the allowable range. The range is the sum of the individual measures with their G. P. E.'s.

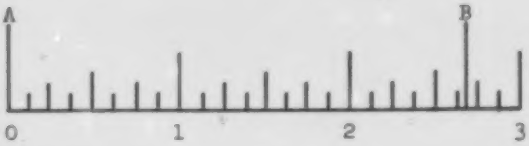
2.3 Greatest Possible Error

The main reason for introducing G. P. E. to the student is to acquaint him with the fact that all measures are approximate and yet when made with care, they do have a maximum or greatest possible error. In everyday living the ability to measure as well as to compute is very important.

Computations with exact numbers give exact answers, e.g., 62 × 4947 = 306,714; but computations with approximate numbers give only approximate answers, e.g., 62" × 4947" could give a great variance in area answer depending upon the size of the unit of measure. If measured in whole inches, then the area could vary from 294,316.75 sq in to 309,218.75 sq in , a difference of 14,902 sq in.

To be consistent, G. P. E. has been defined as $\frac{1}{2}$ the smallest unit of measure used in the measurement. This is a mathematical definition. Of course, students can make much greater errors if they do not know how to measure carefully. The G. P. E. does not guarantee accuracy. Students must still learn how to measure.

Figure 2-3a



In Figure 2-3a:

(A) To the nearest inch $\overline{AB}$ measures 3 inches .

(B) To the nearest $\frac{1}{2}$ inch $\overline{AB}$ measures $2\frac{1}{2}$ inches .

(C) To the nearest $\frac{1}{4}$ inch $\overline{AB}$ measures $2\frac{3}{4}$ inches .

(D) To the nearest $\frac{1}{8}$ inch $\overline{AB}$ measures $2\frac{5}{8}$ inches .

The student's text states the measurement to the city limits sign. Distance between towns on road maps is from City Hall to City Hall. This was done to eliminate confusion on the part of the student.

Exercise 2-3

	Measurement	Smallest Unit	Measurement with G.P.E. Expressed
1.	35 yds., 2 ft.	1 ft.	35 yds., 2 ft. ± $\frac{1}{2}$ ft.
2.	14 ft.	1 ft.	14 ft. ± $\frac{1}{2}$ ft.
3.	14 ft., 3 in.	1 in.	14 ft., 3 in. ± $\frac{1}{2}$ in.
4.	3 meters	1 dm.	3 meters ± $\frac{1}{2}$ dm. or 5 cm.
5.	5 km., 270 m.	1 m.	5 km., 270 m. ± $\frac{1}{2}$ m. or 5 dm.
6.	7 km., 395 m., 47 cm.	1 cm.	7 km., 395 m., 47 cm., ± $\frac{1}{2}$ cm. or 5 mm.
7.	38 miles, 560 yds.	10 yds.	38 mi., 560 yds. ± 5 yds. or 15 ft.
8.	93,562,000 miles	1000 miles	93,562,000 mi. ± 500 mi.

2.4 Measuring Height, Reach and Length of Foot

In choosing the groups the size is not too important, but if a fair comparison of the groups is to be made, there should be, as nearly as possible, the same number in each group.

This could be the student's first exposure to the concept of G. P. E. The activity was purposely kept simple. The authors found that even adults will cooperate when asked to join in the experiment. All seemed interested in their own and their friends' ratio of reach to height.

All student groups need not measure at the same time unless it is convenient to do so in your room. Some could be marking and measuring while others who have finished could be doing the necessary computation.

2.5 Recording the Data

Each student in a group should have a table prepared similar to Table 2-5b.

The decimal equivalents were furnished to aid computing the ratio.

2.6 Computing Ratio

The class might find it interesting to compute the average ratio of reach to height for their group or for the class. The more ratios you average, the closer it might approach 1.00 . The foot size has a greater variation and it probably would gain little if the class average of foot to height were computed.

2.7 Graphing Activity

If the measures of reach and height are rounded off to the nearest inch, graphing the data will be simpler and a closer approximation to a linear function.

If this data is retained by the student or teacher, it will furnish interesting additional material for a class histogram after Chapter 4 is studied.

Exercise 2-7

1. Did the boys or the girls in your row have a closer ratio of reach to height? (Answer will vary.)
2. If you used the smallest and the largest ratio of reach to height in your group, what would be the shortest and longest reach you could expect for a boy 10 feet tall?
   (Answers will vary; should be ≈ 9'7" and 10'6".)
3. A basketball player is 7 feet 3 inches tall. His shoulders are 13 inches from the top of his head, and his arms are 9 inches from the center of his back. How high would you expect him to be able to reach?
   $(87'' - 13'' + (\frac{87''}{2} - 9) = \underline{108\frac{1}{2}''}$ or $\underline{9'\frac{1}{2}''}$.)
4. About how large of a tree trunk (circumference) could the fourth person in Table 2-5b reach around and touch his finger tips? (Answers vary.)
5. If all the students in your row would extend their arms around a larger tree trunk, what would be the largest circumference that they could measure if all fingers were just touching. (Answers vary.)
6. A student's foot measures 10.25 inches from heel to toe. If he walked heel-to-toe for one mile, how many steps would he take? Can this be used as a useful measuring unit? (≈ $6181\frac{1}{2}$ steps; 6181.4634+ ; yes, but probably less accurate than some other devices.)

J.F.F. If a clock strikes 4 times in 3 seconds, how many times will it strike in 9 seconds? (10 - one every second, e.g., 4 in 3 seconds
5 in 4 seconds
6 in 5 seconds
etc.
10 in 9 seconds)

2.8 Addition of Measures

Computation with denominate numbers (a number whose unit represents a unit of measure) can be both interesting and confusing to students. Since all measurements are approximate, we are continuing to suggest that students maintain the idea of precision and greatest possible error.

This section suggests to the student that the idea of the distributive property will also work to explain the addition and subtraction of measures.

Sometimes the fact that $10y + 5y = 15y$ is explained by saying 10 hares + 5 hares = 15 hares. This is satisfactory until we come to $10y \times 5y$; then is 10 hares multiplied by 5 hares equal to 50 square hares? Or even worse, does 5 birds × 7 dogs = 35 bird dogs? If we stick to the mathematical properties of numbers only to justify our position, we are usually on safe ground.

You might want to review the distributive property with the class. Remember, for any numbers a, b and c it is true that $a(b + c) = ab + ac$ and conversely.

2.81 Greatest Possible Error: Addition of Measurement

The greatest possible error of a sum is the sum of the greatest possible errors, and the G. P. E. of a difference is still the sum of the greatest possible errors.

Rules dealing with greatest possible error will cause philosophic problems along with your mathematical problems. The following is just to warn you of the problems, not to cause new ones. Let's use an example. Add $3\frac{1}{4}$", $6\frac{1}{2}$" and 3". Assume the precision is the least unit used in any one measure. One way to add would be as follows:

"A"

$3\frac{1}{4}'' \pm \frac{1}{8}''$

$6\frac{1}{2}'' \pm \frac{1}{4}''$

$\underline{3'' \pm \frac{1}{2}''}$

$12\frac{6}{8}'' \pm \frac{7}{8}''$ or a range of $11\frac{7}{8}''$ to $13\frac{5}{8}''$

If we round off to the unit of least precision before we add, then

"B"

$3''$

$6''$

$\underline{3''}$

$12'' \pm \frac{7}{8}''$ or a range of $11\frac{1}{8}''$ to $12\frac{7}{8}''$

If we add and then round off to the unit of least precision, we have (Rule suggested in text)

"C"

$3\frac{1}{4}'' \pm \frac{1}{8}''$

$6\frac{1}{2}'' \pm \frac{1}{4}''$

$\underline{3'' \pm \frac{1}{2}''}$

$12\frac{3}{4}'' \pm \frac{7}{8}'' = 13'' \pm \frac{7}{8}''$ or a range of

$12\frac{1}{8}''$ to $13\frac{7}{8}''$

If we add, round off the sum to the unit of least precision, and use for the G. P. E. of the sum the number of addens times the G. P. E. of the unit used in the sum, we have

"D"

$3\frac{1}{4}'' \pm \frac{1}{8}''$

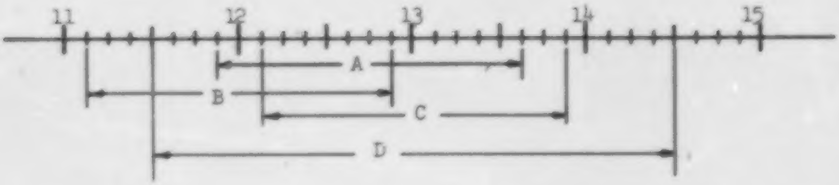
$6\frac{1}{2}'' \pm \frac{1}{4}''$

$\underline{3'' \pm \frac{1}{2}''}$

$12\frac{3}{4}'' \pm \frac{7}{8}'' = 13'' \pm 1\frac{1}{2}''$ or a range of

$11\frac{1}{2}''$ to $14\frac{1}{2}''$

On a number line the ranges would appear as



You will recognize that "C" fits the rules we have chosen to use. "D" might be considered "safer" but does not allow us to use the sum of the G. P. E.

The rules we have suggested in the text seem the most consistent with the rules dealing with significant digits which the student will study in Chapter 3.

Exercise 2-81

1. Perform the following computations:

(A)
$$\begin{array}{r} 37 \text{ mm.} \pm .5 \text{ mm.} \\ + \underline{13 \text{ mm.} \pm .5 \text{ mm.}} \\ 50 \text{ mm.} \pm 1.0 \text{ mm.} \end{array}$$

(B)
$$\begin{array}{r} 41 \text{ cm.} \pm .5 \text{ cm.} \\ + \underline{39 \text{ cm.} \pm .5 \text{ cm.}} \\ 80 \text{ cm.} \pm 1.0 \text{ cm.} \end{array}$$

(C)
$$\begin{array}{r} 64 \text{ ft.} \pm \frac{1}{2} \text{ in.} \\ + 32 \text{ ft.} \pm \frac{1}{2} \text{ in.} \\ \hline 96 \text{ ft.} \pm 1 \text{ in.} \end{array}$$

(D)
$$\begin{array}{r} 3 \text{ yds. } 2 \text{ ft.} \pm \frac{1}{2} \text{ in.} \\ + 9 \text{ yds. } 1 \text{ ft.} \pm \frac{1}{2} \text{ in.} \\ \hline 13 \text{ yds.} \quad \pm 1 \text{ in.} \end{array}$$

(E)
$$\begin{array}{r} 19 \text{ mm.} \pm .5 \text{ mm.} \\ - \underline{17 \text{ mm.} \pm .5 \text{ mm.}} \\ 2 \text{ mm.} \pm 1.0 \text{ mm.} \end{array}$$

(F)
$$\begin{array}{r} 39 \text{ cm.} \pm .5 \text{ cm.} \\ - \underline{38 \text{ cm.} \pm .5 \text{ cm.}} \\ 1 \text{ cm.} \pm 1.0 \text{ cm.} \end{array}$$

Be sure to caution students that even in a subtraction problem (like E and F above) the G. P. E. is added, not subtracted. This might be a good time to take a problem like F and show the various ways it could come out.

(1)
$$\begin{array}{r} 39 \text{ cm.} + .5 \text{ cm.} \\ - \underline{38 \text{ cm.} + .5 \text{ cm.}} \\ 1 \text{ cm.} + 1.0 \text{ cm.} \\ \text{or } 2 \text{ cm.} \end{array}$$

(2)
$$\begin{array}{r} 39 \text{ cm.} + .5 \text{ cm.} \\ - \underline{38 \text{ cm.} - .5 \text{ cm.}} \\ 1 \text{ cm.} \ (\pm 0) \\ \text{or } 1 \text{ cm.} \end{array}$$

(3)
$$\begin{array}{r} 39 \text{ cm.} - .5 \text{ cm.} \\ - \underline{38 \text{ cm.} - .5 \text{ cm.}} \\ 1 \text{ cm.} - 1.0 \text{ cm.} \\ \text{or } 0 \text{ cm.} \end{array}$$

(4)
$$\begin{array}{r} 39 \text{ cm.} - .5 \text{ cm.} \\ - \underline{38 \text{ cm.} + .5 \text{ cm.}} \\ 1 \text{ cm.} \ (\pm 0) \\ \text{or } 1 \text{ cm.} \end{array}$$

2. Find the greatest possible error for the sums of the measurements in each of the following. (Assume that the unit of measurement is the least unit used in any one measure, e.g., $5\frac{1}{2}$" measurement - least unit is $\frac{1}{2}$", G. P. E., $\frac{1}{4}$".)

(A) $5\frac{1}{2}$ in., $6\frac{1}{2}$ in., $3\frac{0}{2}$ in., G. P. E. of sum is ± $\frac{3}{4}$ in. Answer: $\frac{3}{4}$ inch

(B) $3\frac{1}{4}$ in., $6\frac{1}{2}$ in., 3 in. Answer: $\frac{7}{8}$ inch

(C) 4.2 in., 5.03 in. Answer: .055 in.

(D) 42.5 in., 36.0 in., 49.8 in. Answer: .15 inch

(E) 0.004 in., 2.1 in., 6.135 in. Answer: .0510 in.

(F) $2\frac{3}{4}$ in., $1\frac{5}{16}$ in., $3\frac{3}{8}$ in. Answer: $\frac{7}{32}$ inch

3. Add the following measures:
   (A) 42.36, 578.1, 73.4, 37.285, 0.62 Answer: 731.7
   (B) 85.42, 7.301, 16.015, 36.4 Answer: 145.1
   (C) 9.36, 0.345, 1713.06, 35.27 Answer: 1,758.04

4. Subtract the following measures:
   (A) 7.3 - 6.28 Answer: 1.0
   (B) 735 - 0.73 Answer: 734
   (C) 5430 - 647 Answer: 4780

2.82 Greatest Possible Error - "Sum of-computer"

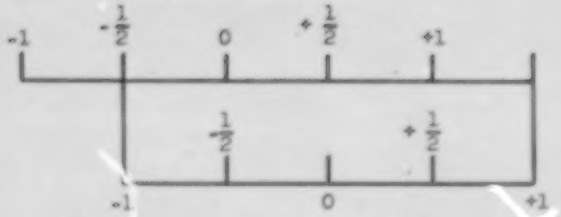
In a subtraction problem of measurements it is possible of course that the G. P. E. may exceed the measure. The idea of the greatest possible error must stress the word greatest. It is the limit of the error. Most errors would be less. The probabilities are rather complex. Students can appreciate the greatest part of the expression if reminded.

In the example in the text,

$$\begin{array}{r} 61 \text{ mm.} \pm .5 \text{ mm.} \\ - \underline{47 \text{ mm.} \pm .5 \text{ mm.}} \\ 14 \text{ mm.} \pm \underline{1.0}\ \underline{\text{mm}}. \end{array}$$

the greatest possible error of ± 1 may cause some discussion. Many students will want to subtract the errors and arrive at zero for the G. P. E. This would be increasing the precision of the measures beyond either of the original measures by a mathematical computation. It is hard to show properly all of the possibilities of combining ± 1 and ± 1 becuase these are only the upper and lower limits. All values in between must also be considered.

A makeshift slide rule might illustrate the idea. See Figure 2-7a in the student text.



Sliding one past the other and noting the sum of the opposing values would demonstrate some values other than the limits. Construct by covering 2 yard sticks or meter sticks with paper marked as in Figure 2-7a. Using marks for -1 to +1 with $\frac{1}{2}$ and $\frac{1}{4}$ marked on it, we can gain an idea of the frequency with which various error combinations might appear.

Sum of 2 G. P. E.	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	
Tallies	|	|	||	||	|||	|||	||||	||||	𝍸	
	1	1	2	2	3	3	4	4	5	Total Number of Tallies 25
Per cent of Total	4%	4%	8%	8%	12%	12%	16%	16%	20%	

Table 2-82a

If the sum computer were completed for values of +2 to -2 by $\frac{1}{4}$ units, the percentages would of course change. However, the concept of "0" being the most prevalent value as against the extremes still holds up.

2.9 Shoes, shoes, shoes ...

The important part of this activity is that the students use several measurements and add them. The sum is checked with one single measurement.

If all measurements are made with care, the sum of the measures with the sum of the G. P. E.'s should make a range which includes the single measure together with its G. P. E.

2.10 Recording Data

The Table 2-10 should be prepared by each member of the group. Its intent is to illustrate the compounding of errors in measurements.

2.11 Summary

Here "growth" is introduced for the first time. Growth curves are dealt with in much greater detail in Chapter 6, where a mold population is used as the activity. Also included in Chapter 6 are data tables to illustrate the uniformity of such "curves" of growth. It is suggested, however, that the drama of this biological phenomenon be saved for that chapter.

Mentioned here are the two concepts of growth: division (multiplication!) of cells and cell specialization. It might be interesting to the students to explain that when cell division takes place in an uncontrolled fashion, cancer results.

An emphasis is placed here on the dynamism of cellular activity. Activity--chemical reactions, division of the DNA molecules, building of protein molecules, even a flame--like action of the cell membrane is going on constantly. Too often as a result of seeing pictures or even when living cells are observed through a microscope, the cells seem to be static, so students think of them as relatively still objects.

Energy is treated more fully in some of the later chapters.

2.12 Optional Activity - Growth from Seeds

This activity is based upon seeds germinating and then showing some growth in both root and stem. In the preliminary version of this text, Chapter 2 was based upon bean plants and their growth, to teach addition of measurement, greatest possible error and levers. From the reports of the teachers who used the preliminary version during the test year, beans planted in containers (milk cartons filled with vermiculite) just do not grow in the wintertime. Schools get cold at night, sunlight is at a minimum and the beans act as though they know it is not spring. For this reason, another activity was chosen to teach the math of addition of measures and G. P. E.

The authors tried the suggested activity of 2.11 during the revision, and we feel that it might work any place. Therefore, it is included as an optional activity which uses the math taught in this chapter.

Preliminary Preparation

Black-eyed peas (cheap if bought by bulk), pumpkin seeds, and watermelon seeds were used in the trial activities. If placed between two wet paper towels in a pie plate for 24 - 48 hours, germination is aided.

The acetate and paper toweling may be cut ahead of time with a paper cutter. Razor blades are best for cutting the milk cartons so you may want to do this yourself or have it done by a selected group.

The straws are glued to the toweling in order to force the plants to grow in 3 parallel rows.

If the seeds are too close to the water (less than 10 cm.), they don't seem to germinate.

The plastic should be cut so that it fits inside the milk carton when placed on the diagonal. Outdated X-ray film may be purchased in packages of 1000 sheets. Your local "chest X-ray" facility may be able to furnish you with some film free.

When all pieces of equipment are ready, the "wafers" may be prepared in class and placed in the containers.

The plant growth will require several weeks. Thus you should plan on continuing on in this or another math book. Have the class check daily for needed water and any plant activity. If the growth is slow, you may want to measure only on the even days.

When the roots have developed and the stems started, a small amount of plant food may be added to the water. It does not take much. Don't overfeed.

The question of root and stem direction is asked of the student. The way the seed is placed does not affect the direction of the growth. This action which requires all roots to go down and the stem up is called geotropism (geo--from the Greek word for the earth). It refers to the response of the parts of the plant to the pull of the earth's gravity.

If the apparatus were turned end for end after a few days of growth, the roots and stem would turn and again grow down and up, respectively.

Experiments have been done with plants growing on a turntable (such as a phonograph) and noting their reaction to the added force. The stems grow against the extra force and lean in, while the roots grow in the direction of the added force.

"Late comers" may be measured but would not ordinarily be considered in a scientific experiment. Science would try to exclude as many variables as possible and therefore would only include those plants germinating at approximately the same time under the same conditions.

We would hope for careful measurements, but again this activity should emphasize that all measures are approximate.

Exercise 2-12a

1. Which day showed the greatest total growth? Answer will vary; that's "life".
2. On which day did your ratio of stem to root change from zero? Answer will vary.
3. Did the ratio $\frac{\text{stem}}{\text{root}}$ ever equal one? Probably. Exceed one? Eventually.
4. Do you think this ratio exceeds 2 for any plant? Yes, some trees, bushes, etc.
5. Is the total growth at a constant rate, that is, the same amount each day? Doubtful.
6. If you had different types of seeds, did each type show the same growth pattern? Doubtful.
7. On which day did you first notice any green coloring? Answer will vary.
8. What does the presence of the green color signify? Answer: The plant now contains chlorophyll and is producing food for its own nourishment (photosynthesis). For more detailed explanation check the biology or botany texts; also notice Chapter 3 of student text.
9. If we used weight rather than length for our measures, what do you think the $\frac{\text{stem}}{\text{root}}$ ratio would be for a carrot, potato and pine tree? Answer: Carrot and potato would probably have a $\frac{\text{stem}}{\text{root}}$ ratio less than 1, while a pine tree, probably greater than 1.
10. Where do the stem and root get their nourishment to grow before the plant foot is added? Answer: From the cotyledon.

Sample Test Items
Chapter 2

1. The __________ __________ __________ in a measurement is __________ of the smallest unit of measure used in the measurement. Answer: greatest possible error; $\frac{1}{2}$.

2. If a box is $23\frac{1}{4}$ in. ± $\frac{1}{8}$ in. in length, we know the measure is between __________ inches and __________ inches. Answer: $23\frac{1}{8}$ inches; $23\frac{3}{8}$ inches.

3. The more precise of two measurements is the one with the __________ possible error. Answer: smaller.

4. Add the following measures:

$3\frac{1}{8}'' \pm \frac{1}{16}''$
$13\frac{1}{4}'' \pm \frac{1}{8}''$
$7\frac{1}{16}'' \pm \frac{1}{32}''$
$8\frac{9}{16}'' \pm \frac{1}{32}''$

Answer: $32\frac{0}{4}$ in. ± $\frac{8}{32}$ in.

(A) What would be the smallest possible measure? Answer: $31\frac{3}{4}$ in.

(B) What would be the greatest possible measure? Answer: $32\frac{1}{4}$ in.

5. Subtract the following measures:

78.7 cm.
- 71.2 cm.

Answer: 7.5 cm. ± .1 cm.

(A) What would be the smallest possible measure? Answer: 7.4 cm.

(B) What would be the greatest possible measure? Answer: 7.6 cm.

6. If Tom is 5 feet 6 inches tall and has a reach of 5 feet 8 inches, what is the ratio of his height to his reach? Answer: 1.03.

7. Betty measured four books and found the following measures:

$6\frac{1}{4}''$, $5\frac{3}{4}''$, $9\frac{3}{8}''$ and $8\frac{1}{8}''$

(A) What was the total measure? Answer: $29\frac{2}{4}'' \pm \frac{3}{8}''$

Dave put the same books on a table and measured them with his steel tape that had $\frac{1}{4}$-inch markings. His answer was 30 inches.

(B) Could Betty and Dave both have been right? Explain your answer.

Answer: Yes, Betty had $29\frac{2}{4}$ in. ± $\frac{3}{8}$ in. and Dave had 30 in. ± $\frac{1}{8}$ in. Betty's greatest measure was $29\frac{7}{8}$ inches and Dave's smallest measure was $29\frac{7}{8}$ inches.

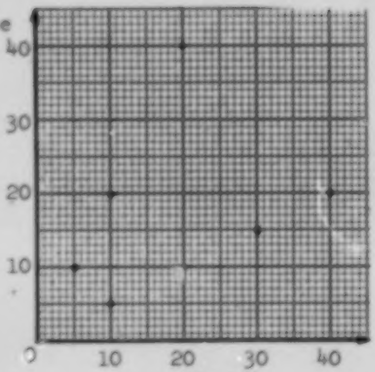
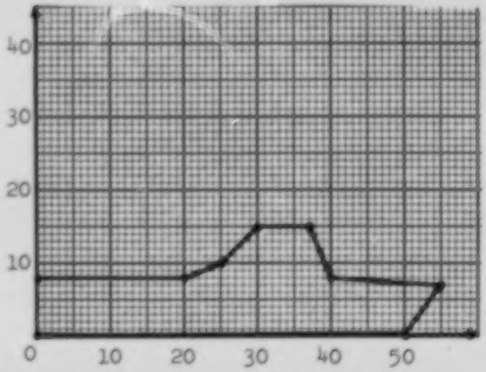
8. In an eighth grade class there are 28 students of whom 16 are girls.
   (A) What is the ratio of girls to the number in the class?
   Answer: $\frac{4}{7}$ or .57
   (B) What is the ratio of the number of girls to the number of boys?
   Answer: $\frac{4}{3}$ or 1.33
   (C) What is the ratio of boys to the number of students in the class?
   Answer: $\frac{3}{7}$ or .43

9. If your height is 64 inches and your shadow is 48 inches long, what is the ratio of your shadow to your height? Answer: $\frac{3}{4}$ or .75

10. What is the ratio of a 12-inch ruler to a yardstick? Answer: $\frac{1}{3}$ or .33

11. The biological term used in this chapter to explain the differences in height, reach, and shoe size is __________ __________. Answer: natural variation.

Sample Test Items
Chapter 2

Graphing ordered pairs	Answer
1. Which of the following sets of numbers could be called an ordered pair? (1, 2, 3 ...), 6.2, .73, (2,5), $35 \pm \frac{1}{2}$	(2,5)
2. If you graphed the ordered pairs (10,14) and (14,10), would they represent the same point?	No
The following question requires a coordinate system.	
3. Given Figure A, plot the pairs (10,5), (20,10), (30,15), (40,20).	 Fig. A
4. On Figure A plot points for the coordinates (5,10), (10,20), (20,40).	
5. On Figure B plot the points for the following ordered pairs. Connect the points with line segments in order as you plot. (0,0), (50,0), (55,7), (40,8), (37,15), (30,15), (25,10), (20,8), (0,8).	 Fig. B

Chapter 3

LEAF SURFACE AREA AND WATER LOSS: AREA, SIGNIFICANT NUMBERS, SCIENTIFIC NOTATION

Teacher's Commentary

3.1 Introduction

Since photosynthesis is discussed here, it might be pertinent to point out a few interesting facts. Light is strongly emphasized (since it is necessary for photosynthesis to take place). However, although the usual source of light is the sun, it need not necessarily be sunlight. Many laboratories use artificial light for their experiments, but intensity is important. Ordinary room light is not usually sufficient. As mentioned in the teacher's commentary of Chapter 1, photosynthesis is the only source of food for living things in the world. Photosynthesis is also the source of most of the oxygen in the air, thereby providing the oxygen needed for respiration (see Chapter 5).

The writers have deliberately avoided being specific in the number of stomates per area of leaf surface, because the leaves used will vary depending on availability. However, if the teacher has access to a microscope, a superb addition to this chapter could be interwoven. See Section 3-8 at the end of the Chapter in the teacher commentary.

Answers to questions.

No, this will not be an accurate measure of the water loss by the entire plant, because any calculations based upon estimates cannot be exact.

3.2 Activity - Leaf Surface and Water Loss

Materials Needed

Large plastic bags (blanket bags), approximately 27" × 36", available in variety stores.

Number needed will depend upon the teacher's decision regarding organization of the class.

This exercise can best be done in teams of three, although it should work if only two bags per class are used, each bag covering a different number of leaves.

In a trial run, the authors chose one branch with 36 leaves and another with 18 leaves. 165 ml of water was collected from the first, while the second yielded 76 ml. This will, of course, be dependent upon the factors indicated in the text. The teacher should survey the school grounds beforehand and carefully select the appropriate tree or trees. Generally, *the larger the leaves the better the results*. (It is possible that the bags may be disturbed by other students. This activity *may* be done at the student's home.)

It is also wise to select a fairly young tree because of the mechanics of *estimating the number of leaves*. This involves counting the leaves in a particular cluster and estimating how many times this size cluster is duplicated on the entire tree. It is suggested that the teacher estimate the number of leaves on the tree before the activity begins.

Procedure

1. Labeling tags should include Team number (if teams are used), class name or number, number of leaves enclosed, date, and hour of placing the bag. Names of team members would be a good idea in case somebody "goofs."
2. Locating branches - see above under *materials needed*.
3. Placing the bags: A word of caution here to the students about injury to the branch would not be out of order. The bag itself will not harm the tree in the 24 hour period of time.

4 and 5. No comment needed.

6. Reiterating the instructions in Chapter 1 - *recording* of data for future reference is extremely important.
7. Some caution is needed here that the tree not be denuded.

Closed curve is discussed in Section 3.3 of the text.

Second Day.

It is suggested that the student store the collected water properly labeled until some concept of volume can be developed.

3.3 Simple Closed Curves

1. Understandings

(a) Broken-line figures such as those we see in statistical graphs, triangles, rectangles, as well as circles, and figure eights are curves.

(b) A simple closed curve in the plane separates the plane into 2 sets - the points in the interior of the curve and the points in the exterior of the curve. The curve itself is contained in neither set (and is itself a third set of points in the plane).

(c) The curve is called the boundary of the interior (or the exterior).

(d) If a point A is in the interior of a curve and a point B is is in the exterior of the curve, then the intersection of AB and the curve contains at least one element.

2. Teaching Suggestions

Draw some curves on the chalkboard, bringing out the idea that we call them "curves" and that a segment is just one kind of curve. We use the word "curve" in a special way in mathematics.

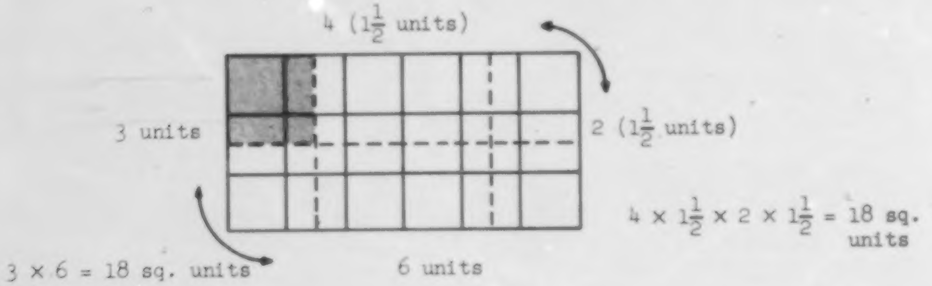
Note that a simple closed curve separates a plane into two sets and that the curve itself is the boundary of the two sets. Also, that any quadrilateral, parallelogram or rectangle is a simple closed curve. Identify some of the many curves which are suggested in the room, such as boundary of chalkboard, total boundary of floor surface, etc.

Students may enjoy drawing elaborate curves which may still be classified as simple closed curves. Encourage their drawing a few simple closed curves for a bulletin board exhibit.

Area of the Region of a simple closed curve - (Rectangles)

In developing the method for computing areas of rectangles from the length and width it is noted that different ways of counting the unit areas illustrate the commutative property of multiplication.

It is hoped that class discussion will bring out the advantages of having a unit of area which is a closed square region, one unit of length on a side. In the case of the illustration of the rectangle 6 units by 3 units, notice the effect of using, as unit of area, a closed square region $1\frac{1}{2}$ units on a side. There is no trouble about covering the closed rectangular region with these units as shown below.


4 ($1\frac{1}{2}$ units)
3 units
2 ($1\frac{1}{2}$ units)
$4 \times 1\frac{1}{2} \times 2 \times 1\frac{1}{2} = 18$ sq. units
$3 \times 6 = 18$ sq. units
6 units

However, the number of squares in each row is no longer the number of linear units, in the length (6), and the number of rows is no longer the number of linear units in the width (3). Thus we would lose the relationship of finding the number of square units of area by multiplying the numbers of linear units in the length and width. It certainly would be possible to devise methods of computing the number of these new square units of area, but it would also be more complicated.

Exercise 3-3

Problems 1-6 are developmental, leading to the method for computing area of a closed curve from the numbers of linear units in the length and width of rectangles added to an estimate of squares not included in the rectangles.

Teacher: These are approximations. Your students may be expected to get answers close to these but not necessarily these same answers.

1.	.92 in^2	4.	1.68 in^2
2.	2.20 in^2	5.	1.91 in^2
3.	2.56 in^2	6.	1.96 in^2

The tracings of the leaf outlines on the students' papers are printed on 10 × 10 per in^2 grids. Hence, the answers are in hundredths in^2.

Obviously there is no attempt made at this point to develop the mathematics necessary for determining the area of an irregular closed curve region. However, your pupils may be interested to know that there are instruments which can be run around the boundaries of a simple closed curve and give at once an approximate value for the area of the closed region. One such instrument is the planimeter, an instrument used by draftsmen and engineers for getting a quick approximation of irregular areas. An important part of the study of calculus, a mathematics course taught at the senior high school or college level, is devoted to finding mathematically the areas of irregular regions to a great degree of accuracy.

3.4 Conversion: Metric - English

The student usually finds problems dealing with the conversion from metric to English (or the reverse) boring and of no practical use. In this unit we have attempted to limit discussion of such conversions to those areas necessary for the solution of the problems without covering all possible cases. Students will undoubtedly raise questions about the number of approximations found in these conversions and this may provide an opportunity to discuss the value of more precise values in the conversions when the measurements they make are only approximations. It should be noted that one unit of English measure, the inch, has been defined in terms of the metric system. One inch is exactly equal to 2.54 cm. Hence any calculations dealing with inches and centimeters may be exact and not approximate. (See Section 3-51.) Hopefully the correct signs = or ≈ are used on all of the conversion tables in the student text.

The students are undoubtedly well aware that most of the world uses the metric system. England determined to change to the metric system in 1965. Canada has partially changed over. The reluctance of the U.S.A is almost solely due to the cost of conversion. General Electric alone estimated a cost of $ 200 million to convert from English to metric.

It seemed most logical to use scientific notation to emphasize the simplicity of the relations involved in the metric system even though scientific notation, as such, is not discussed until Section 3.6.

It is the opinion of the writers that the metric system is most meaningful to the student when learned through usage rather than rote memorization, and numerous exercises in measurements using metric units are encouraged.

3.41 Metric Units of Area

This section includes information used in determining the area of the leaf. From the information given in this section the student should be able to construct a conversion table as follows:

$$1\ \text{in}^2 = 645.16\ \text{mm}^2 = 6.4516\ \text{cm}^2 = 0.064516\ \text{dm}^2 = 0.00064516\ \text{m}^2.$$

These are exact equivalents, not approximates.

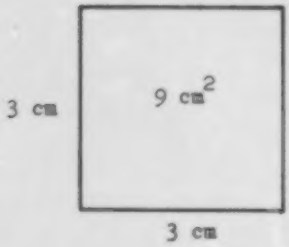
Students should be instructed to justify the decimal placement in this series and compare it with that in the series in the end of Section 3.4.

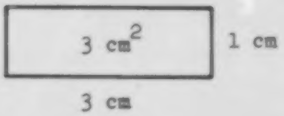
Exercise 3-41

1. How many square millimeters are there in a square centimeter?
   Answer: 100 mm^2 or 10^2 mm^2
2. How many square centimeters are there in a square meter?
   Answer: 10,000 cm^2 or 10^4 cm^2

3. How many square millimeters are there in a square meter?
Answer: 1,000,000 mm^2 or 10^6 mm^2

4. Draw a 3 cm square. Draw also a rectangle whose area is 3 square centimeters. Which is larger?



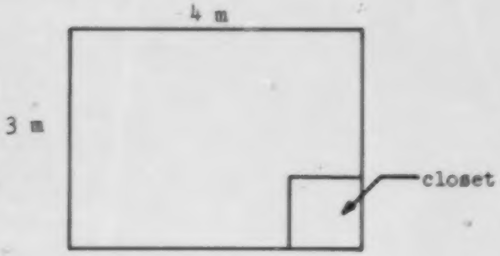


Answer: the 3 cm square is larger.

5. A rug is 2 meters by 3 meters. Find its perimeter and area.
Answer: 2 m + 2 m + 3 m + 3 m = 10 m = perimeter
$2 \text{ m} \times 3 \text{ m} = 6 \text{ m}^2$ = area

6. The floor of a boy's room is in the shape of a rectangle. The length and width are measured as 4 meters and 3 meters. There is a closet 1 meter long and 1 meter wide built into one corner. What is the floor area of the room (outside the closet)?

Answer:



$$\text{Area} = (3 \text{ m} \times 4 \text{ m}) - 1 \text{ m}^2$$
$$= 11 \text{ m}^2$$

7. Conversion of leaf outline areas from Exercises 3-3 to dm^2 using $1 \text{ in}^2 = .064516 \text{ dm}^2$:

1. .0593472 dm^2
2. .1419352 dm^2
3. .16516096 dm^2
4. .10838688 dm^2
5. .12322556 dm^2
6. .12645136 dm^2

Individual students' answers for their own leaf outlines.

3.5 Volume - Metric Units

This unit uses the cubical solid as the metric unit for measuring volume. Using this concept as a beginning it is convenient to show the relationship between volume and capacity (i.e., 1 cc = 1 ml).

Again scientific notation is used to show the simplicity of relationship. The general property of multiplying exponents may or may not be used at this time.

Exercises 3-5

1. Complete each of the following:

   Example: There are $(1000)^3$ or 1,000,000,000 m^3 in 1 km^3.

   (a) There are 10^3 or 1000 mm^3 in a cc.

   (b) There are $(\frac{1}{100})^3$ or $\underline{\frac{1}{1,000,000}}$ m^3 in a cc.

   (c) There are $(\frac{1}{1000})^3$ or $\underline{\frac{1}{1,000,000,000}}$ m^3 in a mm^3

   (d) There are $(10^6)^3$ or 1,000,000,000,000,000,000 mm^3 in a km^3.

2. A rectangular solid has dimensions of 6 cm, 7 cm, and 80 mm. Calculate the volume of the interior of this solid.

   Answer: 6 cm × 7 cm × 8 cm = 336 cm^3

   Recall that the volume of the interior of a rectangular solid is equal to the product of the measures of the length, width, and height, when the measurements are expressed in the same unit.

3. What is the volume of the interior of a rectangular solid whose height is 14 mm and whose base has an area of 36.5 sq cm ?

   Answer: 36.5 cm^2 × 1.4 cm = 51.1 cm^3

3.51 Metric Units of Mass and Capacity

You will notice that without additional comment we have referred to the gram as the unit of *mass*, not weight, in the metric system. It was thought best not to involve the pupils in a full scale discussion of this point here. An adequate treatment of these ideas belongs in a science course. It is likely, however, that some of your pupils will ask questions about this terminology, and you will want to know how to answer them correctly, if not in complete detail. The following discussion should be adequate for this purpose. Should you wish more information on the subject, you might refer to *Physics*, *Volume I* prepared by the Physical Science Study Committee of Educational Services, Inc. (This is the first volume of the so-called M.I.T. course for high school physics students.)

What is the weight of an object? It is a measurement of the force or "pull" of gravity on the object. An ordinary bathroom scale measures this pull by the amount it stretches, or twists, a spring. We think of weight as measuring the "quantity of matter" in an object, in some sense. A box of lead weighs more than the same box filled with feathers because the lead has a greater "quantity of matter" packed into the given volume than do feathers. There is another way to measure the "quantity of matter" of an object. This is to compare the object with some standard, or unit, body on a balance. If we have a supply of identical objects called "grams" we can determine the number of these "grams" it takes to balance the box of lead. This number of grams we call the *mass* of this much lead.

These two different ways of measuring "quantity of matter" can be used interchangeably for most purposes in any one fixed location, but they are not, strictly speaking, measurements of the same thing. Weight depends on the nearness to the center of the earth. The pull of the earth - gravity - on the box of lead would be much smaller in a space ship as far from the earth as is, say, the moon. The weight of the lead would be much smaller there. However, the lead would balance the same number of "grams" on the space ship that it balanced on the earth (the "grams" would themselves *weigh* correspondingly less) so its *mass* would be unchanged. To summarize :

> *Weight* is the measure of the pull of gravity on an object. It decreases as the distance from the object to the center of the earth increases.
>
> *Mass* is a comparison of an object with a set of unit bodies. It does not depend on the position in space where it is measured.

We humans are normally restricted to a very narrow range of altitude above sea level, and, with that restriction, we can think of weight and mass as having a definite fixed relationship. (It is tempting to predict that as we enter the space age and are released from these restrictions, weight and mass and the distinctions between them will become subjects for household discussion.) In the English system weight is measured in pounds, mass in slugs. An object which has a mass of 1 slug has a weight of approximately 32.2 lbs at sea level. The weight in pounds of any object at sea level is approximately 32.2 times its mass in slugs. The more common unit in this system, of course, is the pound. In the metric system, when mass is measured in grams, weight is measured in dynes. An object whose mass is one gram has a weight of approximately 980 dynes at sea level. The weight of this same object in the English system would be approximately 0.0022 pounds. As you probably know, the more familiar unit in the metric system is the unit of mass, the gram.

In the text we have introduced only the more familiar units, pound, and gram. This has made it necessary to use both words, mass and weight. You must judge for yourself how much of the above discussion of the two ideas you will use in your classroom. If the subject does come up, however, be sure to make one point: both mass and weight can be measured in either of the systems of units, English and metric. If you fail to point this out to the pupil, he may interpret the discussion in the text to mean that weight is something measured in the English system and mass something measured in the metric system.

The distinction between mass and weight has to be kept clearly in mind in speaking of the definition of pound in terms of the metric standard kilogram. We say a pound is defined to correspond to 0.45359237 kilograms - that is, the pound (weight) is the weight of a mass of 0.45359237 kilograms. Actually, it is quite correct to speak of a lb (mass) and a lb (weight) so long as the distinction is clearly made. Thus, one may, if he wishes, write :
1 lb (mass) = 0.45359237 kg. We shall not do this here. In the unabridged dictionary, however, a pound is defined as a unit of mass or of weight.

Your classes may be interested in the agreement which went into effect on July 1, 1959, and created (for the first time) an international yard and international pound. The six English-speaking nations (U.S.A., United Kingdom, Canada, Australia, Union of South Africa, New Zealand) agreed to standardize, as of this date, their definitions of yard and pound. The definition of 1 in = 2.54 cm dates from this agreement and so also does the above definition for the pound. Thus, we now have an international yard equal to 0.9144 meters and an international pound corresponding to 0.45359237 kilograms (i.e., the pound weight corresponds to the weight of a mass of 0.45359237 kilograms). Prior to this agreement, the U.S. inch ≈ 2.54005 cm and the British inch ≈ 2.539996 cm, as a result of shrinkage in the British prototype bar. The situation with the pound was even more confused, since, prior to 1959,

1 U.S. lb ≈ 453.5924277 gms,
1 British lb ≈ 453.59233 gms,
1 Canadian lb ≈ 453.59237 gms.

No agreement could be reached for an international gallon. Hence, we still have the U.S. gallon defined as 231 cu in and the British Imperial gallon = 1.20094 U.S. gallons.

Although the metric and the English systems of measure are the major systems, your pupils may be interested in looking in the unabridged dictionary under "measure" to see the great number of other measures used in countries throughout the world.

Exercises 3-51

1. The volume of a jar is 352.8 cc. What is the mass of the water it can contain, expressed in:
   (a) grams? Answer: 352.8 g
   (b) kilograms? Answer: .3528 kg.

2. (a) What is the capacity in milliliters of a rectangular tank of volume 673.5 cc? Answer: 673.5 ml.
   (b) What is its capacity in liters? Answer: .6735 liters

3. A cubical tank measures 6 feet 9 inches each way and is filled with water. (No allowance is made for the approximate nature of measures.)
   (a) Find its volume in in^3. Answer: 531,441 in^3
   (b) Find its volume in ft^3. Recall that 1728 in^3 = 1 ft^3. Answer: 307.5 ft^3
   (c) Find the weight of the water. 1 ft^3 of water weighs about 62.4 lb. Answer: ≈ 19,188 lb.

4. The dimensions of the tank in Problem 3 are about 2 meters each way.
   (a) Find its volume in cubic meters. Answer: $\approx 8.8\ m^3$
   (b) Find its contents in liters. Recall that there are 1000 liters in a cubic meter. Answer: $\approx 8{,}000\,\ell$
   (c) What is the mass of the water? Recall the 1 liter of water has a mass of 1 kilogram. Answer: $\approx 8{,}000$ kg

5. How did the time needed to solve Problem 4 compare with the time needed to solve Problem 3? What is the main advantage of computing in the metric system? Answer: (a) Less time for Problem 4 (b) The decimal relationship of units.

6. A tank has a volume of 2500 cc.

   (a) What is the capacity of the tank in millimeters? Answer: 2500 ml
   (b) How many kilograms of water will the tank hold? Answer: 2.5 kg
   (c) How metric tons of water will the tank hold? Answer: .0025 metric tons

7. A cubical box has edges of length 30 cm.

   (a) What is the volume of the box in cc?
   Answer: 27,000 cc or 2.7×10^4 cc
   (b) What is the capacity in liters?
   Answer: $27\,\ell$ or $2.7 \times 10\,\ell$
   (c) How many kilograms of water will the box hold?
   Answer: 27 kg or 2.7×10 kg
   (assume that it is watertight, of course)

8. The volume of the sun is estimated to be about 337,000 million million cubic miles or 3.37×10^{17} cu miles.
   (a) Using the fact that 1 mile ≈ 1.6 kilometers, express the volume of the sun in cubic kilometers. (Simply indicate multiplications in your answer if you wish.) Answer: $3.37 \times 10^{17} \times (1.6)^3\ km^3$
   (b) Express the sun's volume in cc, leaving your answer in the form of an indicated multiplication.
   Answer: $3.37 \times 10^{17} \times 1.6^3 \times 10^{15}\,cc = 3.37 \times 1.6^3 \times 10^{32}$ cc

9. The British Imperial gallon, used in Canada and Great Britain, is equivalent to 1.20094 U.S. gallons, or

$$1 \text{ British Imperial gal.} \approx 1.2 \text{ U.S. gal.}$$

(a) When you buy 5 "gallons" of gasoline in Canada, how many U.S. gallons do you receive? Answer: ≈ 6 U.S. gallons

(b) How many Imperial gallons are required to fill a barrel which holds 72 U.S. gallons? Answer: ≈ 60 Imperial gallons

3.6 Scientific Notation

Definition: "A number is said to be expressed in scientific notation if it is written as the product of a decimal numeral between 1 and 10 and the proper power of 10. If the number is a power of 10, the first factor is 1 and need not be written." This definition may well need some explanation and many examples. A student may forget and think of numbers between 1 and 10 as only including the counting numbers. Remind him that 1.1 is greater than 1 and can be used in scientific notation.

Exponential notation may need to be reviewed. Practice in changing many large numbers into scientific notation will help all of the students.

Exercise 3-6

1. (a) Is 15×10^5 scientific notation? Why, or why not?
   Answer: No, because 15 is not between 1 and 10.
   (b) Is 3.4×10^7 scientific notation? Why, or why not?
   Answer: Yes, it fits the definition.
   (c) Is 0.12×10^5 scientific notation? Why, or why not?
   Answer: No, because 0.12 is not between 1 and 10.

2. Write the following in scientific notation:
   (a) 5687 Answer: 5.687×10^3
   (b) 14 Answer: 1.4×10
   (c) $3\frac{1}{2}$ million Answer: 3.5×10^6
   (d) 135 Answer: 1.35×10^2
   (e) 14,650 Answer: 1.465×10^4

(f) 35,000,000,000 Answer: 3.5×10^{10}

(g) $7\frac{1}{2}$ billion Answer: 7.5×10^{9}

(h) The product of 50 and 70 Answer: 3.5×10^{3}

3. Write the following in decimal notation:

(a) 3.7×10^{6} Answer: 3,700,000

(b) 4.7×10^{5} Answer: 470,000

(c) 5.721×10^{6} Answer: 5,721,000

(d) 2.25×10^{7} Answer: 22,500,000

(e) 2.8×10^{8} Answer: 280,000,000

(f) $.653 \times 10^{10}$ Answer: 6,530,000,000

(g) 14.75×10^{4} Answer: 147,500

(h) 36.2×10^{3} Answer: 36,200

(i) 386×10^{5} Answer: 38,600,000

(j) $.046 \times 10^{7}$ Answer: 460,000

(k) 6.821×10^{3} Answer: 6,821

(l) $.0038 \times 10^{3}$ Answer: 3.8

4. Since the earth does not travel in a circular path, the distance from the earth to the sun varies with the time of the year. The average distance has been calculated to be about 93,000,000 miles.

(a) Write the above number in scientific notation. The smallest distance from the earth to the sun would be about $1\frac{1}{2}$ per cent less than the average; the largest distance would be about $1\frac{1}{2}$ per cent more than average.
Answer: 9.3×10^{7} miles

(b) Find $1\frac{1}{2}$ per cent of 93,000,000.
Answer: 1,395,000 or 1.395×10^{6}

(c) Find approximately the smallest distance from earth to sun.
Answer: 91,605,000 miles or 9.1605×10^{7} miles

(d) Find approximately the largest distance from earth to sun.
Answer: 94,395,000 miles or 9.4395×10^{7} miles

Some of the more thoughtful students may wonder why we do not write 93 million, for instance, as 93×10^6 where the exponent is used to indicate the number of zeros in the numeral. There is no point in trying to hide the fact that in many cases this is really a little simpler, and there is no reason to try to prevent students from using it. But two things should be made clear: in the first place, this is the notation which the scientists use; and, second, in the use of logarithms and the slide rule, the scientific notation is certainly much more convenient to work with. The ease of making "order-of-magnitude" estimates when the numbers are expressed in scientific notation is perhaps worth mentioning also. A good discussion of orders of magnitude may be found in the PSSC Physics text published by D.C. Heath and Company.

3.7 Significant Digits

The illustration problem under this section is to emphasize the need for some control on the numbers of digits used in a computation. Students are trained through many years of mathematics, that all digits are important and needed. For example, $17932 \times 1457 = 261269244$ if the digits are all significant and not measures or computations taken from other measures.

It is very important for the student to develop an appreciation of the circumstances which warrant extreme accuracy and also when accuracy is less important. The circumference of a circle found by moving a straight ruler around the perimeter, and then using this measurement divided by 3.1415 to find the diameter, correct to 1 / 10,000 of an inch, would be ridiculous.

Some concepts usually contained in a unit on significant digits were not included, not because they were unimportant but because they did not relate directly to the biological experiment. Be sure the student has an opportunity later when circumstances permit to have an exposure to relative error, percent of error, and accuracy.

Continually remind the students that all measurements are approximate regardless of the instruments used. We can refine the measurement, we can reduce the possible error, and we can increase the precision, but we are still approximating.

Any study on the proper use of significant digits always seems to have some discrepancies. To further complicate it, different texts and math lectures give different definitions for significant digits and their use. What we can do in this study is just to agree with the students to see the need for some control and to be consistent in some set of "rules." Other rules have been tried and they all seem to have as many problems as these proposed. Don't suggest the difficulties to the students but be sure and work with the student when and if they arrive.

Computations on approximate measurements cannot increase the accuracy. The measurement 5264 inches is not as precise as the measurement $1\frac{1}{2}$ inches because the second measurement is to the nearest $\frac{1}{2}$ inch, a smaller unit. The measurement 375 feet is more precise than the measurement 2 yards because the first measurement is in the smaller units.

It is possible that a measurement of 1950 feet might be significant to four places, if the measurement was to the nearest foot and not the nearest 10 feet as it seems to indicate. In this case a small bar may be placed under the zero to indicate that it is to be included in the number of significant digits. This bar is not a vinculum which is a straight bar placed over quantities to indicate the same operation as parentheses, brackets, or braces.

Exercise 3-7

(1)	8	1
(2)	.01	1
(3)	457	3
(4)	3.32	3
(5)	3005	4
(6)	36.70	4
(7)	$67\underline{00}$	3
(8)	23.000	5
(9)	620.03	5
(10)	.0028	2

J.F.F. If a bottle and its cork cost two and a half cents, and the bottle alone costs two cents more than the cork, what is the cost of the cork?

Answer: $\frac{1}{4}$ ¢ Let X = cost of the cork; then X + 2 ¢ = cost of bottle

Therefore, X + (X + 2 ¢) = total cost or 2.5 ¢

$$2X + 2¢ = 2.5¢$$

$$2X = .5¢$$

$$X = \frac{1}{4}¢ = \text{cost of cork}$$

3.71 Multiplying and Dividing Measurements

This section emphasizes the need for some control of numbers found in measurements. Reasonableness must be used by the students. The guide for multiplying measures suggests that the product of two numbers should not contain more significant digits than the number of significant digits in the less accurate factor.

Some students may well suggest, "If I measure a width of a section in tenths of an inch, why cannot the area be correct to tenths of an inch?" Let's try one.

width: 50.6 " length: 70.3 " area: ___?___

If we multiply 50.6" × 70.3" we have 3557.18 sq in or to the nearest tenth 3557.2 sq in. This seems reasonable but - using our information of G.P.E., we have

$$\left(50.6 \pm \frac{1}{20}\right)\left(70.3 \pm \frac{1}{20}\right) =$$

$(50.6 + .05)(70.3 + .05)$ = maximum area of 3563+ sq in

$(50.6 - .05)(70.3 - .05)$ = minimum area of 3551+ sq in

12 sq in difference which is a long way from the scant tenth of a inch.

$\frac{1}{20}$" evenly spread out along 70" is a lot of area.

Be sure when two factors have an unequal number of significant digits that the rounding off of the greater one is done before the multiplication is done. Then the product also must be rounded off to the same number of significant digits as the factor having the least number of significant digits.

Exercise 3-71a

1. Multiply the following approximate numbers:

   (a) 4.1×36.9 Answer: 150

   (b) 3.6×4673 Answer: 17,000

   (c) $3.76 \times (2.9 \times 10^4)$ Answer: 11×10^4 or 1.1×10^5 in scientific notation

2. Divide the following approximate numbers:

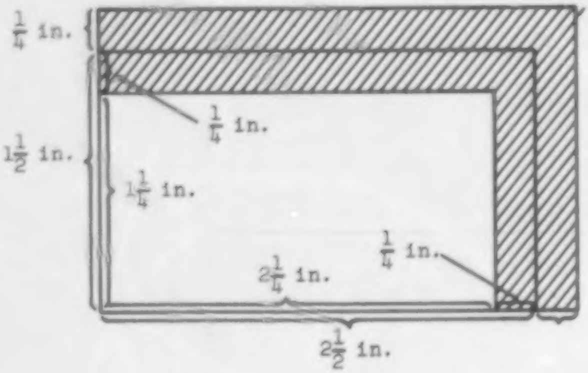
   (a) $3.632 \div 0.83$ Answer: 4.4

   (b) $0.000344 \div 0.000301$ Answer: 1.14

   (c) $(3.14 \times 10^6) \div 8.006$ Answer: 3.92×10^5

3. Suppose a rectangle is $2\frac{1}{2}$ inches long and $1\frac{1}{2}$ inches wide. Make a drawing of the rectangle. Show on the drawing that the length is ($2\frac{1}{2} \pm \frac{1}{4}$) inches and the width ($1\frac{1}{2} \pm \frac{1}{4}$) inches. Find the largest area possible and the smallest area possible, and find the difference or uncertain part. Find the area using the measured dimensions, and find the results to the nearest $\frac{1}{2}$ square inch.

Answer:
Largest area = $4\frac{13}{16}$ sq in
Smallest area= $2\frac{13}{16}$ sq in
Difference = 2 sq in
Area is ($3\frac{3}{4}$ sq in) or 4 sq in (to the nearest $\frac{1}{2}$ sq in)



4. Find the area of a rectangular field which is 835.5 rods long and 305 rods wide. Answer: 2.55×10^5 rods^2

5. The circumference of a circle is stated $C = \pi d$, in which d is the diameter of the circle. If π is given as 3.141593, find the circumferences of circles whose diameters have the following measurements:
(a) 3.5 in Answer: 11 in
(b) 46.36 ft Answer: 145.6 ft
(c) 6 miles Answer: 20 miles

6. A machine stamps out parts each weighing 0.625 lb. How much weight is there in 75 of these parts? Answer: 46.9 lbs

7. Assuming that water weighs about 62.4 lb per cu ft, what is the volume of 15,610 lbs of water? Answer: 250 ft^3

J.F.F. Suppose the distance around the equator is 25,000 miles and that the surface is quite smooth and circular in section. If a steel band made to fit tightly around it is then cut, and a piece eighteen feet long welded into it; how loose will the ring be? In other words, what will be the size of the gaps all around between the inside of the ring and the earth's surface? Could you slip a piece of paper under it? Crawl under it?

Answer: Approximately 3' above the earth all the way around.

One solution $C = \pi d$ or $\frac{C}{\pi} = d$

Substitute the values: $\frac{25{,}000}{\pi}$ miles = diameter of the earth

$\frac{25{,}000 \text{ miles} + 18 \text{ ft}}{\pi}$ = diameter of the ring with addition

$\frac{25{,}000 \text{ miles}}{\pi} + \frac{18 \text{ ft}}{\pi}$ = diameter of the ring

$\downarrow \qquad \searrow$

(dia. of earth) + (6 ft) = diameter of the ring

The ring would be approximately 3' above the earth if evenly spaced.

Exercises 3-71b

1. Take the leaf outline areas from Problem 7, Exercise 3-41 which you converted to dm^2 units and round them off to the correct number of significant digits.

Answers:

(1) .059 dm^2
(2) .142 dm^2
(3) .165 dm^2
(4) .108 dm^2
(5) .123 dm^2
(6) .126 dm^2

2. If the leaf outline enclosed approximately 180 squares on a graph paper with $\frac{1}{4}$ inch squares, calculate its area in square decimeters.

Answer: $\frac{180}{16} \approx 11$ in^2

Answer with two significant digits $= 11 \times .065 \approx .72$ dm^2

3. (a) If the approximate area of a leaf were 210 quarter inch squares what would be the area in square decimeters?

   Answer: $\approx .85$ dm^2

   (b) Express the answer to (a) in square centimeters.

   Answer: ≈ 85 cm^2

(c) Express the answer to (a) in square millimeters.
Answer: ≈ 8500 mm^2

4. If a leaf were outlined on graph paper with $\frac{1}{2}$ inch squares and the approximate area was computed at 85 squares, what would be the approximate area in square decimeters? Answer: ≈ 1.38 dm^2

5. These answers are dependent on the students' individual leaf outlines.

Exercise 3-71c

In this exercise students are given step-by-step instructions for finding the theoretical amount of water transpired by the tree(s) selected for the experiment. The calculation is based on the theoretical average of 9.6 g/gm^2 of water transpired in a 24 hour period. Next the students are instructed to calculate the actual transpiration for a 24 hour period and the calculation is based on the measurements made by the students.

When comparing their answers (line 10 to line 5) they will probably find a difference. To account for any difference they will have to recall the discussion earlier in this chapter that explains some of the factors relating to transpiration. The main factors that affect transpiration are (1) amount of water in the soil, (2) the temperature, and (3) the amount of light (sunlight) that strikes the tree in the 24 hour period.

Exercise 3-71d

A young (2 year) mulberry tree was found to have approximately 400 leaves with an average surface area of 1.3 square decimeters. One reference gives the average number of stomates per cm^2 (lower side only) on mulberry leaves as 48,000. Let us assume that this tree has an average water loss of .5 grams per dm^2 per hour. Solve the following. (Express answers in scientific notation.)

1. The number of stomates per dm^2. Answer: ≈ 4.8×10^6 stomates
2. The number of stomates per average leaf.
   Answer: $4.8 \times 10^6 \times 1.3 \approx 6.2 \times 10^6$ stomates
3. The approximate number of stomates on the tree.
   Answer: $6.2 \times 10^6 \times 400 \approx 2.5 \times 10^8$ stomates

4. The water loss per day from an average leaf.

   Answer: $.5 \times 1.3 \times 24 \approx 15.6$ g

5. The water loss per day by the tree.

   Answer: $15.6 \times 400 \approx 6240$ or 6.24×10^3 g

6. The water loss per year (365 days) by the tree (in grams).

   Answer: $6.24 \times 10^3 \times 3.65 \times 10^2 \approx 2.28 \times 10^6$ g

7. The number of liters of water lost by the tree in one year.

   Answer: Assume 1 ml weighs 1 g. Then:

   $$2.28 \times 10^6 \text{ g} = 2.28 \times 10^6 \text{ ml} = 2280 \, \ell$$

8. The number of gallons of water lost by the tree in one year.

   Answer: 1 gal ≈ 3.785 liters; therefore:

   $$\frac{2280 \; \ell}{3.785 \; \ell \, / \, \text{gal}} \approx 602 \text{ gal}$$

3.8 An Optional Activity - Counting Stomates

If the teacher has a microscope available, the following technique could be most effectively used. The area of the field of the microscope can be readily estimated by placing a transparent metric ruler on the stage and manipulating until the approximate center of a mm mark is at the edge of the field, then estimating the position of the opposite edge of the field. Most commonly used microscopes (Spencer or Bausch and Lomb) usually measure 1.5 mm (1500 microns) at 100 magnification. By counting the number of stomates included in this area, the student should be able to estimate the actual total number of stomates on his leaf, and use this figure in his calculations, instead of the average given.

Another thing which could be done would be a further calculation of the actual amount of water per stomate transpired by the leaves included in the plastic bag. This is an excellent opportunity for further use of scientific notation, since amounts might well range in the region of 10^{-6} to 10^{-9}.

Materials

Microscope
Slide
Cover slip
Leaf from the tree being studied

A leaf is opaque, and since light must be passed through any object being studied by the usual light microscope, therefore a thin, transparent piece of the tissue must be obtained.

Procedure

Place a drop of water in the center of the slide.
When one tears a leaf, usually one finds a very thin, transparent layer of tissue extending from the main body of the underside of the leaf.
This need not be more than two mm on a side for use here.
Place this piece of epidermis in the drop of water on the slide.
Tilt the cover at an angle over the drop of water and allow it to "fall" on the drop of water. The tissue is now ready for examination.
The stomates will appear as tiny "mouths." (Two semi-circular cells will be joined at both ends, resembling lips, with an opening between.)

Sample Test Items

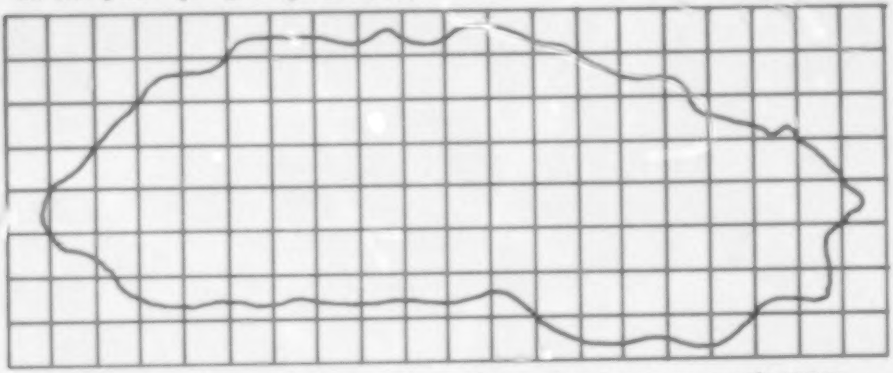
Matching

ans.
1. (d) produced by photosynthesis
2. (h) means 1,000,000
3. (i) ten meters
4. (g) connects stem to root in bean plants
5. (c) green pigment in plants
6. (b) stem of a leaf
7. (j) means $\frac{1}{1,000,000}$
8. (a) must have chlorophyll and light
9. (e) loss of water by a plant
10. (f) openings in the leaves

(a) photosynthesis
(b) petiole
(c) chlorophyll
(d) sugar
(e) transpiration
(f) stomates
(g) hypocotyl
(h) mega
(i) dekameter
(j) micro

A small tree has approximately 5000 leaves with an average surface area of 2.8 square decimeters. The average number of stomates per square centimeter on leaves of this tree is 20,000. Answer the following questions if the tree has an average water loss of .3 grams per dm^2 per hour. (Give answers in scientific notation.)

	Answers
1. The number of stomates per dm^2.	1. 2×10^6
2. The number of stomates per average leaf.	2. 5.6×10^6
3. The approximate number of stomates on the tree.	3. 2.8×10^{10}
4. The water loss per day from an average leaf.	4. 8.4 grams
5. The water loss per day by the tree.	5. 4.2×10^3 grams
6. The water loss per year (365 days) by the tree (in grams).	6. 1.533×10^6 grams
7. The number of liters of water lost by the tree in one year (1000 grams = 1 liter).	7. 1.533×10^3
8. The number of gallons of water lost by the tree in one year (1 gal ≈ 3.785 ℓ)	8. 405.019 gal



If the simple closed curve shown above were drawn on graph paper (4 squares to the inch), what would be the area of its interior?

6 sq. in

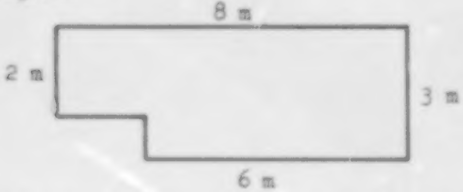
Changing metric measures	Answers
1. $\frac{1}{10}$ meter is ______ decimeters.	1. 1
2. 1 meter is ______ centimeters.	2. 100
3. 1 gram is ______ milligrams.	3. 1000
4. 385 millimeters is ______ meters.	4. .385
5. 31 dekameters is ______ centimeters.	5. 31000

1. If one inch is defined as exactly equal to 2.54 cm, what is the exact metric equivalent of 1 in^2 ?
   6.4516 cm^2

2. How many square centimeters are in one square meter?
   10,000 cm^2

3. There are 1,000,000 mm in one square meter. Express the number in exponential form and in terms of a meter.
   10^{-6} m or $\frac{1}{10^6}$ m

4. What is the area in meters of the following diagram?
   22 m^2



5. The area of a large sycamore leaf is 270 cm^2. Express this area in dm^2.
   2.70 dm^2

6. A rectangular solid has dimensions of 5 cm, 50 cm, and 500 cm. What is the volume in cm^3 ?
   125,000 cm^3

7. A rectangular solid has dimensions of 70 mm, 6.5 cm, 1.7 dm. What is the volume in m^3 ?
   .00007735 m^3

8. What is the mass in grams of the water contained in a jar which will hold 964.5 cc.?
   964.5 g

9. Which weighs the most: 57 grams of water or 57 grams of lead?
   same

10. Which has the greater mass: 135 milliliters of gasoline or 135 milliliters of gold?
    gold

11. Is 13×10^7 a number expressed in scientific notation?
    No

12. Write the equivalent of 59,400 in scientific notation.
    5.94×10^4

13. Express the distance to the moon (238,000 miles) in scientific notation.
    2.38×10^5 miles

14. Express the distance of a round trip flight to Mars in scientific notation (assume straight lines and enter Mars at its closest position to earth : 34,600,000 miles).
    6.9×10^7 miles

15. If a rocket ship could travel at the average speed of 25,000 miles per hour, how long would it take it to make a round trip to Venus? (Assume Venus is 26,000,000 miles from Earth.) Give answer in terms of hours and in scientific notation. 2.08×10^3 hrs

16. Express 2.93×10^6 in decimal notation. 2,930,000

17. (a) How many significant digits are there in 2973? 4
    (b) How many significant digits in .0038? 2
    (c) How many significant digits in 900.5? 4
    (d) How many significant digits in 94.00? 4
    (e) How many significant digits in 0.003? 1
    (f) How many significant digits in $94\underline{0}0$? 3

18. 4.1 and 61.37 are approximate numbers. Using our rules for multiplying approximate numbers, find the answer and express in the correct number of significant digits. 250

19. Let $\pi \approx 3.1416$. Find the circumference of a circle with a diameter of 2 ft. Give the answer in the correct number of significant digits. 6.2 ft

Chapter 4

MUSCLE FATIGUE: PERCENT; MEAN, MEDIAN AND MODE; INFORMAL EXTRAPOLATION; HISTOGRAM

Teacher's Commentary

4.1 Introduction

A very brief introductory statement on muscle fatigue is used as a reminder of a fact that students will be well aware of. What they may not be so aware of is the fact that a simple exercise using only arm muscles, done under the instructions as given, can rapidly tire the muscles used. Let the students "discover" this.

4.2 Counting vs. Measurement

The teacher might well think of other examples to emphasize the difference between counting and measuring. Some excellent and historic examples are described in SMSG's Mathematics for Junior High School, Volume 1, Part 1, pages 21 and 67.

4.3 Activity 1 - Muscle Fatigue

A watch or clock with a second hand is necessary for timing purposes.

It is most important that the students be cautioned to follow instructions exactly. Touching the table and the palm of the hand with the finger tips every time is essential. Some good-natured policing will undoubtedly be necessary. (It was necessary even among adults!)

If a student shows an increase in number per 30 seconds, he is not doing it properly.

Be sure the instructions are thoroughly preread and that the students understand the activity before beginning.

The rest period should not allow complete "recovery", so the fourth time period should read higher than the third but not as high as the first.

The muscles fatigued need not be identified by name. They should, however, be identifiable by location in the arm. Muscles on the anterior (palm) side of the forearm should be noticeably tired from closing the fingers, while those on the back of the arm are used to open the fingers. However, during trial runs, noticeable fatigue was felt up to the shoulder, indicating that a number of muscles are involved.

4.4 How do muscles work?

Answers to questions:

If both ends of a muscle were attached to one bone, no motion could take place unless the strength were so great that the bone was broken. Muscles cannot "push". They can only contract, then relax, thereby allowing the opposing muscle to lengthen them again.

Flexing (bending) the elbow is done by the biceps muscle in the front of the upper arm. Its points of attachment are at the upper end of the humerus (upper arm bone) and just below the elbow on the upper end of the radius (one of the two lower arm bones). Extending the elbow is accomplished by the triceps muscle at the back of the upper arm. Its points of attachment are the shoulder blade and the ulna (the other lower arm bone).

Heart muscles are not discussed here because of space. Food and oxygen are carried to the cells by the blood stream, and wastes are removed also by the blood stream.

It is hoped that the students can reason the above answers from their experiences. This inquiry (discovery) technique, the writers feel, is most important in the encouragement of logical reasoning.

4.41 Interpreting Data

The graphs constructed from the sample data show a definite fatigue in muscles and a slight recovery with rest. The students' graphs will vary considerably. If the exercise was done correctly, fatigue will show and probably recovery will not. You will notice that the size and apparent strength of the student did not necessarily correlate with the data and graph. Some girls may "outdo" the class athletes.

4.5 Percent

The concepts of percent are introduced briefly. The meaning of percent is based on the idea that "x%" means $\frac{x}{100} = x \times \frac{1}{100}$. Notice that the solutions of all problems are set up in the form:

$$\frac{a}{b} = \frac{x}{100}.$$

The method of solving for x should be the method that the pupil understands. Pupils should be encouraged to use any method they understand.

In the example of the class of 11 girls and 14 boys, the fraction $\frac{44}{100}$ indicates percent easily because the denominator is 100. The sum of $\frac{11}{25}$ and $\frac{14}{25}$ is 1. The sum of the ratios $\frac{44}{100}$ and $\frac{56}{100}$ is 1, and the sum of 44% and 56% is 100% or 1.

In the proportion $\frac{14}{25} = \frac{c}{50} = \frac{d}{75} = \frac{56}{100} = \frac{e}{125} = \frac{f}{150}$, $c = 28$, $d = 42$, $e = 70$, $f = 84$.

Exercise 4-5

1. Write each of the following numbers as a percent.

(A) $\frac{1}{2}$
(B) $\frac{1}{4}$
(C) $\frac{3}{4}$
(D) $\frac{1}{5}$
(E) $\frac{2}{5}$
(F) $\frac{3}{5}$
(G) $\frac{3}{2}$
(H) $\frac{2}{2}$
(I) $\frac{4}{4}$
(J) $\frac{7}{5}$
(K) $\frac{5}{4}$
(L) $\frac{5}{2}$

Answers

(A) 50%
(B) 25%
(C) 75%
(D) 20%
(E) 40%
(F) 60%
(G) 150%
(H) 100%
(I) 100%
(J) 140%
(K) 125%
(L) 250%

2. Consider the following group of students.

Student	Hair Color
William	blond
Joanne	brown
Mary	blond
John	red
Joe	brown
Betty	brown
Roy	blond
Don	brown
Margaret	red
David	brown

(A) What percent are boys? 60%

(B) What percent are blonds? 30%

(C) What percent are redheads? 20%

(D) What percent are not redheads? 80%

(E) What percent are brown-haired girls? 20%

(F) What percent are redheaded boys? 10%

J.F.F. A man bought a radio for \$40, sold it for \$60, purchased it back for \$70, and finally sold it for \$90. How much money did he gain or lose on the complete transaction?

Answer: Cost \$40 + \$70 = \$110. Income \$60 + \$90 = \$150. Difference \$40 profit.

4.51 Applications of Percent - Increase and Decrease

Solutions to percent are based on the fact:

$$\text{if } \frac{a}{b} = \frac{c}{d}, \text{ then } ad = bc.$$

(Multiply both sides by bd and reduce.) It may be necessary in some classes to review this. If the students discover valid short cuts for themselves, they should be encouraged to use them if they understand what they are doing. A little space is given to checking by estimation. Some teachers may want to stress these further.

Exercise 4-51

Applications of Percent - Increase and Decrease

1. A junior high school mathematics teacher had 176 pupils in his classes. The semester grades of the pupils were 20 A's, 37 B's, 65 C's, 40 D's, 14 F's. Find, to the nearest tenth percent, the percent of grades that were A's, the percent that were B's, and so on.

(A) A = 11.4%
(B) B = 21.0%
(C) C = 36.9%
(D) D = 22.7%
(E) F = 8.0%

What is the sum of the answers in parts (A), (B), (C), (D), and (E)? Answer: 100%. Does the sum help to check the answers? Yes. The sum of percents must add up to 100.

2. Bob's weight increased during the school year from 72 pounds to 81 pounds. What was the percent of increase? Answer: $12\frac{1}{2}\%$.

3. During the same year, Bob's mother reduced her weight from 140 pounds to 126 pounds. What was the percent of decrease? Answer: 10%.

4. On the first of September John's mother weighed 130 pounds. During the next four months she decreased her weight by 15%. However, during the first four months of the next year, her weight increased 15%. What did she weigh on the first of May? Did the answer come out as you expected?

   Answer: A little over 127 pounds. One should expect this, since the 15% which is added is computed on a smaller amount than the 15% which is subtracted.

   130 pounds × .85 = 110.5 pounds   110.5 × 1.15 = 127.05 pounds ≈ 127 pounds

4.6 Mean - Median - Mode - Measures of Central Tendency

This section introduces the concept of average or arithmetic mean to point out a problem often associated with tables of data that contain a large number of items. Many scientific experiments require data from a large number of trials and then the finding of an average to represent the data of all the trials. Median and mode are briefly discussed as a logical follow-up of the discussion of mean.

Exercise 4-6

1. Find the mode of the following list of chapter test scores:
   79, 94, 85, 81, 74, 85, 91, 87, 69, 85, 83. Answer: 85

2. From the scores in Problem 1, find the

   (A) Mean $(\frac{913}{11} = 83)$ (B) Median (85)

3. The following annual salaries were received by a group of ten employees:

   $4,000; $6,000; $12,500; $5,000; $7,000;
   $5,500; $4,500; $ 5,000; $6,500; $5,000.

   (A) Find the mean of the data. Answer: $6,100.
   (B) How many salaries are greater than the mean? Answer: 3.
   (C) How many salaries are less than the mean? Answer: 7.
   (D) Does the mean seem to be a fair way to describe the typical salary for these employees? Answer: No! The mean is larger than such a large percentage of the salaries.

(E) Find the median of the set of data.

Answer: $\frac{\$5,000 + \$5,500}{2} = \$5,250$

(F) Does the median seem to be a fair average to use for this data?
Answer: Median is better than mean, since the mean gives the impression that the salaries are higher than they are.

4. Following are the temperatures in degrees Fahrenheit at 6 p.m. for a two-week period in a certain city: 47, 68, 58, 80, 42, 43, 68, 74, 43, 46, 48, 76, 48, 50.
Find the (A) Mean, (B) Median.
Answer: Mean = 56.5; Median = 49.

4.7 Informal Extrapolation

Mathematical extrapolation of values can be rather sophisticated. The intent in this section is to very informal. The "idea" of extrapolation is to be introduced, not the involved mathematics. Pupils often project or extrapolate their mathematical ideas. Students also often interpolate and may ask about the difference between interpolation and extrapolation. The difference between the two is: inter-polation is a process of finding values inside the range of the function, while extra-polation is the process of finding values greater than or less than the known values of the function.

The first example in the student text sets up the feeling of a sequence.

1st floor	5'		
2nd floor	15'	diff. of 10'	(2 × 10) - 5
3rd floor	25'	diff. of 10'	(3 × 10) - 5
4th floor	35'	diff. of 10'	(4 × 10) - 5
8th floor	75'	4 times 10'	(8 × 10) - 5
150th floor	1495'	142 times 10'	(150 × 10) - 5

The series 2, 4, 6, and 8 would give a sixth term of 12. If a trackman could run

100 yards in 10 seconds		
200 yards in 22 seconds	diff. of 12 sec.	
300 yards in 36 seconds	diff. of 14 sec.	diff. of 2 sec.
400 yards in 52 seconds	diff. of 16 sec.	diff. of 2 sec.
500 yards in 70 seconds	diff. of 18 sec.	diff. of 2 sec.
600 yards in 90 seconds	diff. of 20 sec.	diff. of 2 sec.

The warning pertaining to too little data for extrapolation is important. Using two terms, such as 2 and 4, is dangerous. Two such terms can lead to all kinds of third terms.

The use of the log tables was just to illustrate that extrapolation can at times be involved and might lead to misconceptions. The log of 600 is 2.77815.

Chapter 5, Section 5.6, covers the subject of extrapolation and interpolation more completely.

Exercise 4-7

Solutions to Problems 1 through 12 in this exercise are based on data in the student's tables (Table 4-3a), and answers will vary.

13. What does your graph illustrate as an answer to our original question, "Does exercise cause fatigue and does one recover quickly?"
    Answer: The graph should show that exercise does cause fatigue and rest gives some recovery.

14. Did you "recover" fully in the 30-second rest period?
    Answer: Most of the students should not recover fully (achieve the same number in the fourth time period as in the first) if they did the exercise conscientiously.

15. What differences, if any, were apparent in the right and left hands? How do you account for the differences?
    Answer: The hand that is used most might show less fatigue. However, many factors might influence results: carelessness, ambidexterity, much practice at the piano, etc.

16. How might a physical therapist use data of this kind to be able to better help his patient? Class discussion.
    Answer: Various exercises exert different muscles. A physical therapist uses his knowledge of physiology to suggest activities which will strengthen the proper muscles.

17. Could you "feel" where the muscles were located that contracted to close the fingers? Open them?
    Answer: The muscles which contract to close the fingers are located on the palm side of the forearm. Those that open the fingers, on the back side of the forearm. In this exercise muscles up to the shoulder are used, so some students may complain of tiredness in the upper arm.

18. If class time permits, or at home, "re-do" the experiment, except do not stop until you have completed six or more consecutive 30-second intervals. Graph your results in another color on your present graph.

The suggestion that the experiment be repeated without a rest interval can be done at the teacher's discretion. Some will undoubtedly try at home. It would tend to lend credence to their extrapolation.

4.8 A Histogram

This is a concept hard to define with simple words but not too difficult to use.

To avoid embarrassment, you may desire to have each student write his "loss" on a slip of paper. If this is not a problem, they may be read out loud in the class by each student.

A histogram made of a large quantity of data will approximate a normal distribution. One class may not offer enough different bits of information. Data from several classes could be combined.

If the class tends toward several distinct occurrences, a second histogram may be constructed, based on smaller counts on the horizontal reference line. When a large number of students fall within the range 15 to 20, for instance, the units may be as shown in Figure 4-8a below.

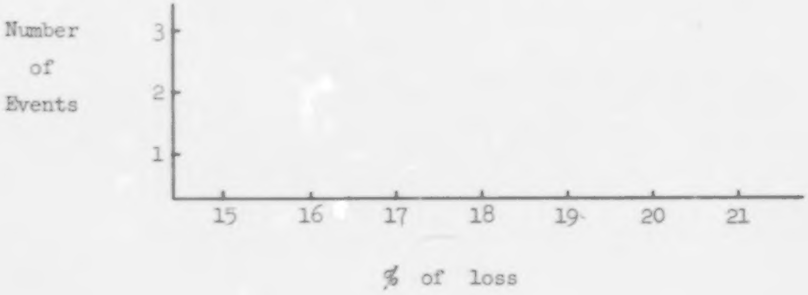


Figure 4-8a

An example of a histogram made from combined data may appear as the one below (Figure 4-8b), where data from 14 individuals is entered. The percent of students showing the same range of percent of loss is written at the top of each frequency.

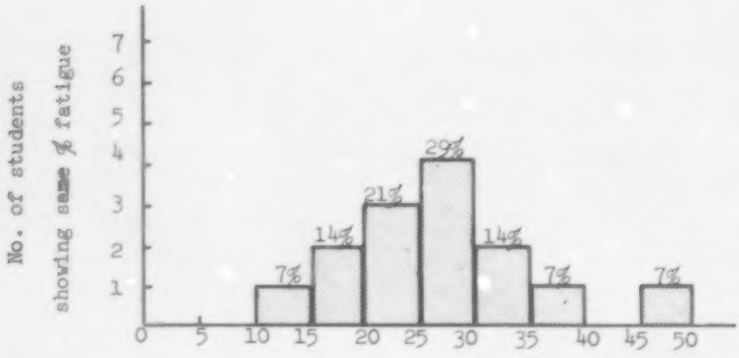

Percent of Fatigue

Figure 4-8b

An easy way for students to find the percentages to be written at the top of each range in the histogram is to find the reciprocal of the total number of students. Using our example (Figure 4-8b), in which data from 14 students is entered, we will find the reciprocal of 14. Then $\frac{1}{14} = 0.071$, so

$$1 \times .071 = 7\%$$
$$2 \times .071 = 14\%$$
$$3 \times .071 = 21\%$$
$$4 \times .071 = 29\%.$$

If many problems are to be done with the same division, then the use of a reciprocal will aid in the computation. Books of instruction accompanying calculators will suggest such a process. Some of the electronic computers change division problems into multiplication problems by the use of reciprocals.

Additional data for a class histogram can be found in Chapter 2. The students were requested to save their data which will now make another activity histogram $\left(\frac{\text{Reach}}{\text{Height}}\right)$.

4.9 Optional Activity - Exercise and Pulse Rate

Here an attempt is made to show the nature of the statement of a scientific problem. Students (and other inexperienced questioners) are often prone to stating a question in such a say that it is nearly impossible to answer.

Breathing rate would be more difficult because the students often try very hard to *control* the breathing, thus decreasing the validity of the data.

Since this has been suggested as an optional exercise, the teacher may wish to do this as a class demonstration, using two or three students only and recording the data on the board.

There may be some question in regard to all students participating if there is any possibility that there are heart problems or other physical restrictions. It is suggested that the teacher *check with the school nurse* to be safe. If possible, invite the school nurse to the classroom for instruction in pulse taking.

It is also suggested that boxes or platforms of *step height* be used if the entire class (including girls) is to participate. The rate should return to *nearly* normal in two minutes.

One suggestion--on the day before the activity request the girls to wear *full* skirts the following day.

Exercise 4-9

Questions 1 through 6 will depend on *individual* data.

The answer to Question 7 will be found in Section 4.4 in the text.

Questions 8 through 11 will depend on *individual* data.

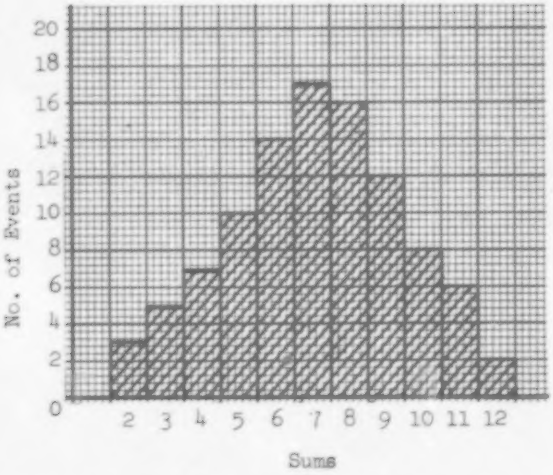
Questions 12 and 13 depend on *class* data.

14. Could you say then that the normal rate of increase is known? Why?
    Answer: No, "normal" would depend upon a much larger sampling and would be determined by standardized procedures, age, sex, weight, height, metabolic rate, etc.

Sample Test Items

		Answers
1.	In the following set of numbers what is the mean? (1, 1, 1, 2, 4, 5, 7)	3
2.	In the set (4, 9, 8, 9, 5, 1, 2, 2) what is the mean?	5
3.	In the set (9, 5, 11, 12, 4, 3, 13, 1, 8) what is the median?	8
4.	Find the mean, median, and mode of the following set of numbers: (1, 1, 3, 4, 6, 6, 7, 8, 9, 11, 11, 11, 13)	mean 7 median 7 mode 11
5.	What is the mode of the set in Problem 1?	1
6.	What is the median of the set in Problem 2?	$4\frac{1}{2}$
7.	A junior high school math teacher had 186 pupils in his classes. The semester grades were 22 A's, 37 B's, 74 C's, 38 D's, 15 F's. Find to the nearest tenth of a percent the percent of grades that were B's.	19.9%
8.	Given the set of numbers (2, 4, 6, 8, 10 ...), extrapolate for the 8th term.	16
9.	Given the set of numbers (1, 1, 2, 3, 5, 8, 13, 21 ...), extrapolate for the 9th term.	34
10.	Given the set of numbers (... 18, 21, 24, 27), extrapolate for the term just before 18.	15
11.	Given the set (5, 14, 9, 8, 12, 3, A, 15, 2, 7, 9, 5, 12, 7, B, 10), 45% of the mean is 3.6; the mode is 7. What numbers are represented by A and B in the set?	3,7
12.	Construct a histogram to represent the following data. Two hexahedrons (dice) were rolled 100 times. The sums were noted as follows:	

Sum of	No. of times
2	3
3	5
4	7
5	10
6	14
7	17
8	16
9	12
10	8
11	6
12	2



True, False

(T) 13. In measurements you can never say exactly a certain number.

(T) 14. In counting you can say exactly a certain number.

(F) 15. You use the same muscles to open your hand as you do to close it.

(F) 16. Muscles "push" as well as "pull".

(T) 17. When a muscle contracts, it shortens and thickens.

(T) 18. When muscles are attached to bones, there is always at least one joint between the muscle connections.

(T) 19. Skeletal muscles are arranged in opposing pairs.

(F) 20. Visceral muscles are the muscles one of the pairs of which is attached to bone and the other is not.

(F) 21. Food and oxygen are carried from the muscles to the lungs by the blood.

(F) 22. The Latin phrase per centum means "by the thousand."

A class of 28 students has 12 boys and 16 girls. One boy and two girls have red hair.

23. What percent of the class are boys? 43%
24. What percent of the class has red hair? 11%
25. What percent of the class are girls with red hair? 7%
26. What percent of the class are girls? 57%
27. The one boy with red hair is what percent of the class? 4%
28. What percent of the class does not have red hair? 89%

Consider the following group of students:

Student	Hair Color
Barbara	black
Mary	blond
Harry	blond
Betty	brown
David	brown

29. What percent are boys? 40%

30. What percent of the girls have black hair? 20%

31. The population of a city was 53,000 in 1956. In 1965 its population was 113,000; what percent was this of the 1956 population? 213%

32. If the town mentioned in the last problem had a population of 104,000 in 1970, what percent would this be of the 1965 population? 92%

Chapter 5

YEAST METABOLISM: LINEAR GRAPHING, CURVE FITTING, EXTRAPOLATION AND INTERPOLATION, VOLUME OF A CYLINDER

Teacher's Commentary

5.1 Introduction

An attempt is being made here to point out the unifying biological themes that are being used for these activities: natural variation, growth, metabolism, adaptation to environment, and the similarities in all living things (unity).

There are, of course, many examples which teachers can glean from students' experiences which will recall to them the body's requirement for faster heartbeat and deeper breathing after strenuous exercise.

The questions are simply for speculation and to lead the thinking into the difficulties inherent in answering some of the questions to which one might want the answers.

5.2 What is Respiration?

The teacher may wish to clarify for the student the two uses of the term respiration. The common usage, i.e., artificial respiration, respiration rate, refers to what is in this chapter called breathing. The biochemical interpretation of the term--oxidation of food (usually carbohydrate) to release energy, carbon dioxide, and water in the cells--is the interpretation that is being used here. This is a further emphasis on the unity of pattern in living things.

Students might ask:

1. How do insects breathe? They have small openings along the sides of the abdomen, called spiracles. These lead into tubules which penetrate to all parts of the body.
2. Spiders have "book lungs," leaf-like structures which offer a large surface area to the atmosphere.
3. Many water-living organisms have gills, which strain the water, removing the dissolved oxygen (not from the H_2O molecule). This is why

goldfish, etc., need a fairly large surface area in the bowl.

4. Tadpoles have gills; frogs and toads have lungs; hence amphibia.

5. Whales and other water mammals must surface occasionally to breathe air.

6. Simple birds (which have a very high rate of metabolism) have an extremely fast breathing rate as well as heartbeat.

7. In general, the smaller the animal the higher the metabolic rate.

Anaerobic organisms (those that live in the absence of oxygen) are for the most part microscopic, since this is a relatively inefficient way of producing energy. However, all cells are capable of temporary anaerobic existence. The process described in the text is often called the "oxygen debt." There are many degrees of existence varying from the higher vertebrates, which are quite aerobic (the central nervous system suffers quickly from oxygen attack), to completely anaerobic organisms, such as some parasitic protozoans (one-celled animals) and certain types of bacteria. Clostridium botulinum is a bacteria which cannot metabolize in oxygen and, incidentally, is the cause of the infamous food poisoning "Botulism." Yeast is an example of an organism which can live well in either circumstance. If held under anaerobic conditions, it utilizes sugar to produce alcohol and carbon dioxide. When oxygen is available, it oxidizes the alcohol completely to carbon dioxide and water.

5.3 Activity - Measuring Yeast Metabolism

Materials needed:

1. Disposable plastic syringes (without needles), one per team. These are usually available from your family doctor or a nearby hospital, if they have a clear understanding of your purpose. If available, the 12 ml. syringe is far preferable to use than a smaller one.

2. Clear plastic tubing to fit the syringe nozzle tightly. Intravenous (I. V.) tubing is also usually readily available from hospitals, if one talks diplomatically to the nurses. Most I. V. tubing comes with rubber fittings which can be adjusted to fit the nozzle. Aquarium tubing may also fit. You will need approximately 18 inches for each team.

3. 12-inch plastic millimeter ruler, preferably with a groove down the center. This could be improvised, of course.

4. Yeast, either dry or cake. Mix the yeast culture by adding approximately $\frac{1}{2}$ package (or $\frac{1}{2}$ cake of yeast) and $\frac{1}{2}$ cup of ordinary sugar to about 2 cups of warm water.

   This should be done at least 20 minutes before the first class, to give the yeast time to become active.

   This culture should last all day, although afternoon readings may be slower, in spite of somewhat higher temperatures.

5. Sugar--ordinary table sugar ($\frac{1}{2}$ cup).

6. Food coloring. Red is easily visible. Add 2 or 3 drops to 3 or 4 drops of water.

7. Transparent tape or masking tape.

8. Block with white paper. The purpose of this is to allow manipulations (connecting and disconnecting) of the tubing and syringe nozzle. Any improvised gadget will work.

   White paper--two reasons.
   a. Attachment for tape to anchor the apparatus in place.
   b. Visibility of drop and ruler markings.

9. Clock or watch for measuring one-minute intervals.

Procedure

It might be wise to have the students prepare their tables in advance, perhaps as a homework activity.

1. The teacher may find it desirable to set up the apparatus for each team before class to save time. Certainly all nozzle-tubing connections should be checked before issuing.

2. You may also wish to premix the dye and provide it in separate small vials or bottles, along with the caps from milk cartons.

3-4-5. Partly fill (less than half full) the syringe with yeast culture. You may wish to do this and have it taped to the block before class starts.

6. This will have to be done by the students, as action should begin immediately after attachment.

   If the drop does not move, the usual explanation is a "leak" somewhere for gas to excape. Check fittings. If not fittings, yeast may be stale.

Also check for any blockage that might have inadvertently occurred.

7. It is conceivable that the drop could move so fast you would have to cut the time to 30-second intervals.

Temperature variation. Yeast should metabolize faster as the temperature rises--roughly twice as fast with each 10° C. increase. There is, of course, a top limit when the yeast would slow, then die.

The reason for a second reading (even a third if you can manage the time) is explained in the text. (See Section 5.5.)

Answers to Questions

The moving drop can be explained only by gas production. However, if by chance the apparatus slopes downhill, the students might suggest this interpretation. For this reason, the apparatus should be level, if not slanted slightly uphill.

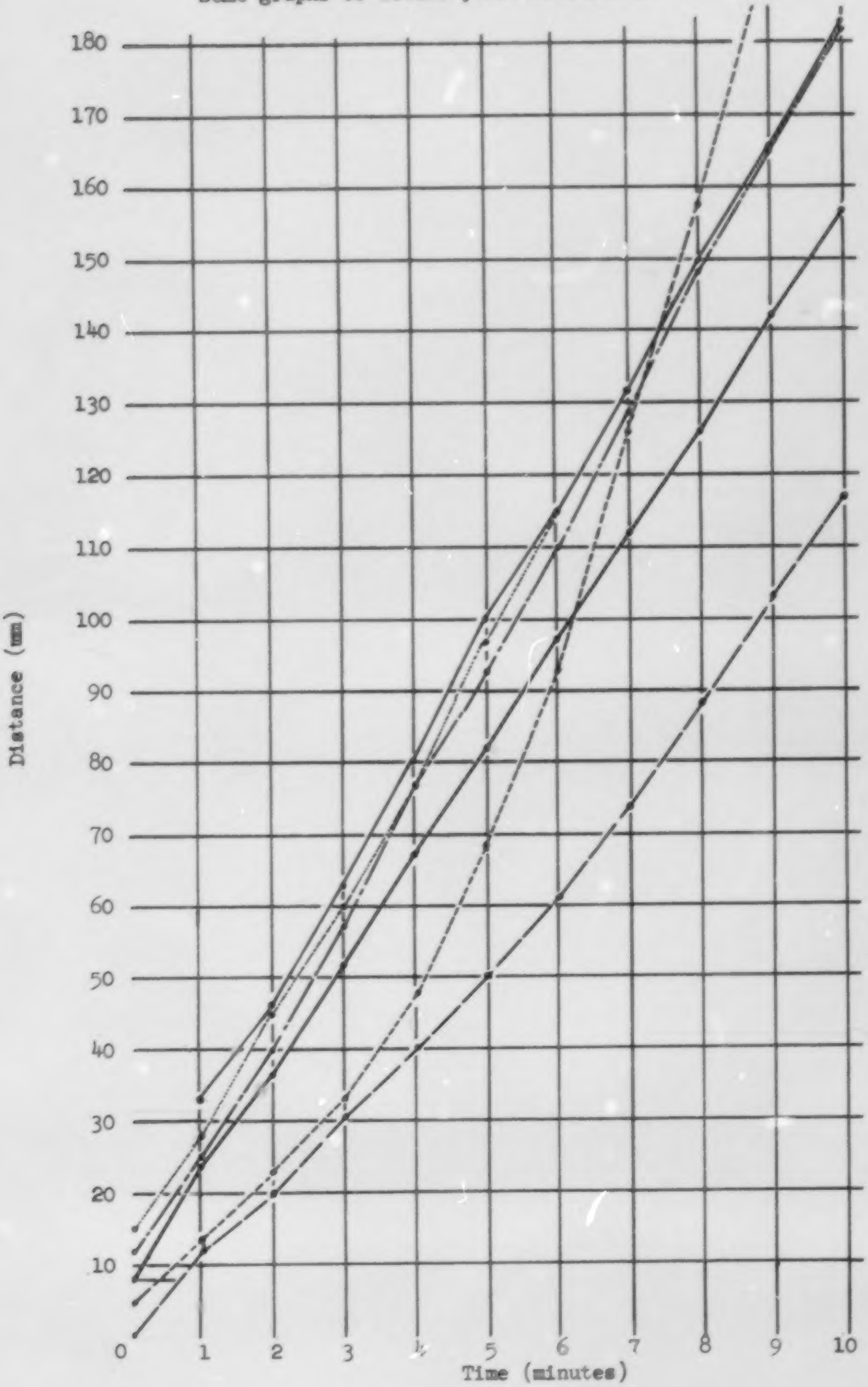
The students are asked later in the chapter to calculate how much gas is produced per minute. At this time, the teacher should encourage them to think through for themselves possible methods of doing this.

The graph should show a linear rate. This is the major reason for working with this activity. (See graphs at end of chapter.)

5.4 Graphing the Data

This type of experiment is done to illustrate a physical phenomenon which gives definite points of reading and yet generates a line. The concept of points between points to justify a line is not completely valid of course. An infinite number of points so placed would still leave gaps for other points, but the concept of density establishes the notion of a line. The readings are themselves not exact because the drop of liquid was moving (generating a line) all of the time.

Some graphs of actual yeast metabolism.



5.5 Curve Fitting

Many students will be familiar with Zeno's Paradox, but it is introduced here anyway because of its loose analogous reference to points of time related to space. If a student believes the paradox, then he must believe that the man in front in any race will win, regardless of the speed of the other contestants.

To refute the paradox, it is a good idea to introduce a time factor. If Achilles can run 1000 yards in, say, 5 minutes, then the tortoise can run 100 yards in 5 minutes. At the end of 10 minutes, Achilles will have run 2000 yards, the tortoise, 200 yards, and Achilles will be 800 yards ahead.

If a student wonders "where" Achilles catches up, you might use this simple algebraic solution.

Since they start at the same time and we "stop" them when Achilles catches up, each runs the same length of time. Thus, $t = t$. If the tortoise goes x yards before being caught, then whatever "x" is, Achilles runs $x + 1000$ yards. Putting this information into a table, we have:

	Achilles	Tortoise
rate	1000 yards per unit of time	100 yards per same unit of time
time	t	$= \quad t$
distance	$x + 1000$ yards	x yards

If $d = rt$, then $t = \frac{d}{r}$. For each case the relation would be:

Achilles: $\frac{x + 1000}{1000} = t$; Tortoise: $\frac{x}{100} = t$;

but the times are the same. Thus, equating the equations, we have one equation with one unknown.

$$\frac{x + 1000}{1000} = \frac{x}{100}$$

$$\frac{1000(x + 1000)}{1000} = \frac{x(1000)}{100}$$ Multiply both sides by 1000

$$x + 1000 = 10\,x$$ Simplify

$$1000 = 9\,x$$ Combine terms

$$\frac{1000}{9} = x$$ Divide both sides by 9

$$111.1111 \approx x$$

When Tortoise runs ≈ 111.111 yards, Achilles runs ≈ 1111.111 yards, and he catches the tortoise.

The curve fitting for the graph is just to be a graphical notion of what we all do anyway. We tend to fit things into patterns. Our points appear as a pattern in this graph. The "curve" is just to graphically finalize the pattern. You might explain there are several very involved formulas to use in curve fitting. These require a knowledge of math beyond the scope of this grade level.

5.6 Interpolation and Extrapolation

Prior to this chapter the students have been asked to do some informal extrapolation. The method of proportional parts for interpolation and extrapolation is presented in this section. A second method uses observation of the plotted points to estimate the best position of the line.

Interpolation by proportional parts is a good drill exercise in the use of proportion for a practical purpose. A set of math tables will furnish you with large amounts of data. Many tables are not linear and thus if you call for too great extrapolation, the error will be considerable. Math tables often include a section on proportional parts to aid in the interpolation.

If possible, give your students other opportunities to interpolate. As they advance through school, they will have many more occasions to use the knowledge.

Exercise 5-6a

From Figure 5-6b and line B, answer the following:

1. A width of 40 mm. would give an expected length of ≈ 48 mm.
2. Find the expected length for a leaf width of 15 mm. ≈ 20 mm.
3. If the width is 45 mm., the expected length would be ≈ 54 mm.
4. If the length is 45 mm., the expected width would be ≈ 38 mm.
5. Where does line B intersect the axis? ≈ 3
6. What would be the mathematical interpolation of your answer to Problem 5?

   A leaf with a length of 3 mm. would have a width of zero mm.

7. Would the information you have about biology help you to justify your answer to Problem 6? Why?

Yes. It can't happen. Any leaf would have some length and width!

Exercise 5-61a

(Expanded solutions on the following page)

Use the data from the following table to answer the questions in this exercise.

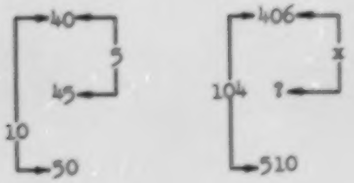
First Term	Second Term
10	100
20	201
30	303
40	406
50	510
60	615
70	721
80	828
90	936
100	1046

1. What would be the second term for the ordered pair (30, ?)? 303
2. What would be the first term for the ordered pair (?, 721)? 70
3. Find the second term for the first term of 45. 458
4. Find a second term for the ordered pair of (63, ?). 647
5. Find the second term for a first term of 87. 904
6. Find the first term for a second term of 252. 25
7. Find the first term for a second term of 980. 94
8. The first term is 110. Find the second term. 1156
9. The first term is 103. Find the second term. 1079
10. The second term is 836. Find the first term. 81

Expand solutions to the preceding exercise (5-61a).

1. 303
2. 70
3. 458

First term **Second term**


40
5
45
10
50
406
x
?
104
510

$\frac{5}{10} = \frac{x}{104}$; $\frac{1}{2} \cdot 104 = x$

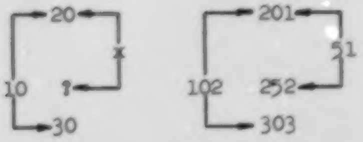
$x = 52$; add $52 + 406 = \underline{458}$.

4. 647 $\frac{3}{10} = \frac{x}{106}$; $x = 31.8$ or 32; $615 + 32 = \underline{647}$.

5. 904 $\frac{7}{10} = \frac{x}{108}$; $x = 75.6$ or 76; $828 + 76 = \underline{904}$.

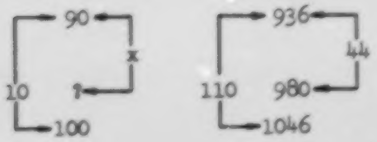
6. 25 252 is between 201 and 303. Thus, the first term is between 20 and 30.

First term **Second term**


20
x
?
10
30
201
51
252
102
303

$\frac{x}{10} = \frac{51}{102}$; $x = 5$; First term is $20 + 5 = \underline{25}$.

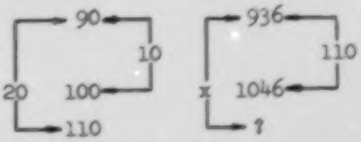
7. 94


90
x
?
10
100
936
44
980
110
1046

$\frac{x}{10} = \frac{44}{110}$; $x = 4$;

$90 + 4 = \underline{94}$.

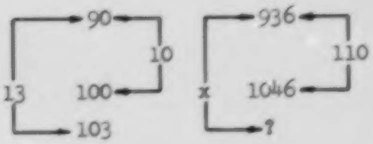
8. 1156 **Extrapolate**


90
10
100
20
110
936
110
1046
x
?

$\frac{10}{20} = \frac{110}{x}$; $10x = 2200$; $x = 220$

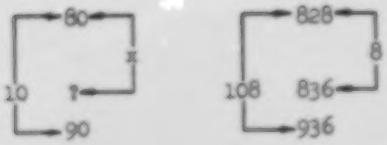
$936 + 220 = \underline{1156}$.

9. 1079


90
10
100
13
103
936
110
1046
x
?

$\frac{10}{13} = \frac{110}{x}$; $x = 143$; $936 + 143 = \underline{1079}$.

10. 81



$$\frac{x}{10} = \frac{8}{108}; \quad x = .7 \approx 1; \quad 80 + 1 = \underline{81}.$$

Exercise 5-61b

1. If the condition of your experiment did not change, how far would you expect the mark in the tube to advance by the end of one hour? (Pretend the tube is of sufficient length.)

   Answer: Depends on student data.

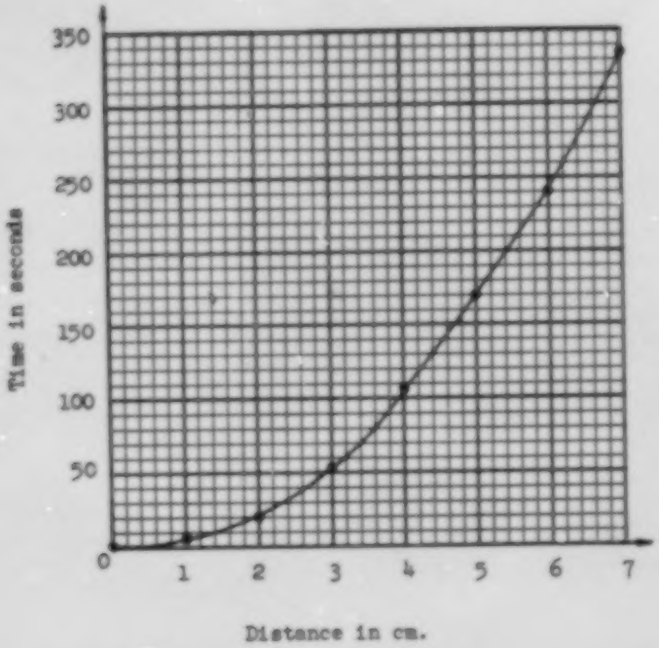
2. Under the same conditions as in Problem 1, what distance could you anticipate by the end of one week?

   Answer: Depends on student data.

3. Given a tube of infinite length, do you think the action shown in your experiment would continue for one hour? One week? One month? One year? Why? Can be used as a discussion question.

   Answer: Under room conditions, certainly an hour. Action would probably cease prior to one week. Typical population explosion problem. Overcrowding uses up food supply, etc.

4. Below is a graph of data gathered in a wick experiment. A strip of chromatography paper was calibrated in centimeters, dipped in a jar of water, and time readings were taken as the water moved up each centimeter of length.



(A) Each small square on the d-axis represents what part of a centimeter?

Answer: .2 cm.

(B) How far did the water move in the first minute? (Give answer in tenths of a centimeter.)

Answer: About 3.2 cm.

(C) How far did the water move in the first 20 seconds?

Answer: About 2 cm.

(D) How long did it take the water to move up to the 5-centimeter mark?

Answer: About 170 seconds.

(E) How long did it take the water to move up $2\frac{1}{2}$ centimeters?

Answer: About 35 seconds.

(F) The smooth curve in this graph indicates a continuous function. What does this mean to you regarding values in between circled points?

Answer: The movement of the water up the wick was a continuous action, so all points on the graph line are meaningful.

5.7 Volume

This section is included to serve as a review in calculating volumes. The method of calculating the volume of the rectangular solid is used to point out the fact that volumes can be calculated using measurements of length if the units are multiplied to result in units cubed. It is often confusing to pupils to talk of cubic centimeters (cc or cm^3) and milliliters as both being a measure of volume. When capacities are discussed, it should be pointed out that a cylinder such as a tube has a capacity determined by the internal diameter (I.D.), and care should be taken in these measurements. It is easier to measure the outside diameter (O.D.), and this often turns out to be the measurement taken.

Exercise 5-71a

1. Find the volume of a rectangular object that is 4 cm. wide, 8 cm. long, and 2 cm. high.
   Answer: 64 cm^3

2. Given a bottle with flat sides (planes), a square base, and the inside edges of the base 1 cm., what would be the volume of liquid when filled to a depth of:
   (A) 1 cm. Answer: 1 cm^3
   (B) 2 cm. Answer: 2 cm^3
   (C) 4 cm. Answer: 4 cm^3

3. Given another bottle with flat sides but with a square base of 2 cm. on each internal edge, what would be the volume of liquid when filled to a depth of:
   (A) 1 cm. Answer: 4 cm^3
   (B) 3 cm. Answer: 12 cm^3
   (C) $\frac{1}{2}$ cm. Answer: 2 cm^3
   (D) $2\frac{1}{2}$ cm. Answer: 10 cm^3

4. Given a metal box (parallelopiped), rectangular base with internal measurements of 4 cm. for length, 2 cm. for width, and 25 cm. high, what would be the volume of liquid when filled to a depth of:
   (A) 1 cm. Answer: 8 cm^3
   (B) 2 cm. Answer: 16 cm^3
   (C) 5 cm. Answer: 40 cm^3
   (D) 25 cm. Answer: 200 cm^3

Exercise 5-7a

1. Information is given for five right cylinders. The letters r and h are the measures of the radius of the circular base and the height of the cylinder, respectively. Using 3.1 as an approximation for π, find the volume of each cylinder.

   The numerical answers obtained when computing volumes of cylinders sometimes appear more precise than the indicated accuracy of the original measurements would justify. It is reasonable to round some of the answers.

	Cylinder	Radius (r)	Height (h)	Volume
(A)	A	4 in.	8 in.	400 in^3

Answer: $V \approx 3.1 \times 4^2 \times 8 \approx 396.8 \text{ in}^3 \approx 400 \text{ in}^3$

(B)	B	8 ft.	4 ft.	790 ft^3

Answer: $V \approx 3.1 \times 8^2 \times 4 \approx 793.6 \text{ ft}^3 \approx 790 \text{ ft}^3$

(C)	C	10 cm.	30 cm.	9300 cm^3

Answer: $V \approx 3.1 \times 10^2 \times 30 \approx 9300 \text{ cm}^3$

(D)	D	7 yd.	25 yd.	3800 yd^3

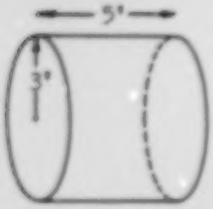
Answer: $V \approx 3.1 \times 7^2 \times 25 \approx 3937.5 \text{ yd}^3 \approx 3800 \text{ yd}^3$

(E)	E	12 in.	12 in.	5400 in^3 or 3.1 ft^3

Answer: $V \approx 3.1 \times 12^2 \times 12 \approx 5356.8 \text{ in}^3 \approx 5400 \text{ in}^3$

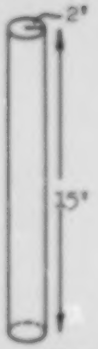
2. Find the volumes of the right cylinders shown here. The dimensions given are the radius and the height of each cylinder. The figures are not drawn to scale. (Use $\pi = 3.1$.)

(a)



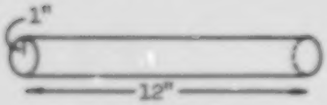
about 140 cu ft

(b)



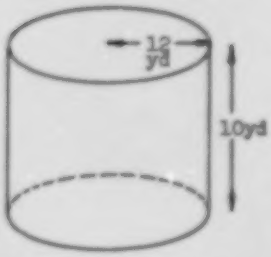
about 186 cu ft

(c)



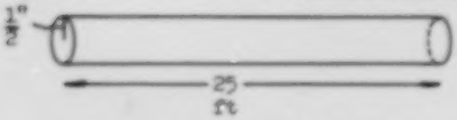
about 37 cu in

(d)



about 4464 cu yds

(e)



about 232 cu in or about 0.13 cu ft

3. A silo (with a flat top) is 30 feet high and the inside radius is 6 feet. How many cubic feet of grain will it hold? (What is the volume?) Use $\pi \approx 3.14$.

   Answer: $V \approx 3391.2$ cu. ft.

4. A cylindrical water tank is 8 feet high. The diameter (not radius) of its base is 1 foot. Find the volume (in cubic feet) of water which it can hold. Leave your answer in terms of π. If you use 3.1 as an approximation for π, what is your answer to the nearest (whole) cubic foot?

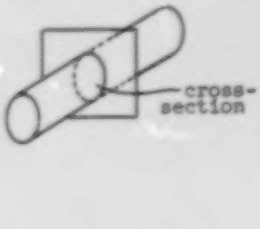
   Answer: Volume is 2π cu. ft., $\approx$ 6 cu. ft.

5. There are about $7\frac{1}{2}$ gallons in a cubic foot of water. About how many gallons will the tank of Problem 4 hold?

   Answer: $\approx$ 47 gallons (or 45 gallons if computed from 6 cu. ft.)

6. Find the amount of water (volume in cubic inches) which a 100-foot length of pipe will hold if the inside radius of a cross-section is 1 inch. Use $\pi \approx 3.14$. (A cross-section is the intersection of the solid and a plane parallel to the planes of the bases and between them.)

   Answer: $V \approx 3768$, or volume is about 3770 cu. in.



J.F.F. A man walked one mile south, then one mile east, another one mile north, and found he had returned to his starting point. Where did he start if he was south of the equator?

Answer: Many students would say the North Pole. This is one starting point but not south of the equator. Actually there is an infinite number of starting points. They all lie on a circle near the South Pole. There is a circular path around the world exactly one mile long ($\approx \frac{1}{3}$ of a mile north of the South Pole). If the man started one mile north of this path then he could walk one mile south, one mile east (taking him around the world), and one mile north returning him to his starting point.

5.8 Volume of Gas Produced by Yeast Activity

The data and questions asked in this section are all based upon an actual experiment. It is used here just prior to asking the students to compute volumes in their own experiment. One experiment (done by a biology instructor) used the following equipment:

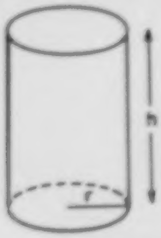
Total capacity of syringes = 12 cubic centimeters (cc)
Diameter (inside measure) of tubing = 3 mm.
When started, the syringe contained 5 cc of liquid and
7 cc of gas (air).

If the tube is a cylinder with a diameter of 3 mm., what is the area of the base of such a cylinder? Answer: $\approx 7\ mm^2$

How many mm^3 are needed to make $1\ cm^3$? Answer: 1000

If you poured $1\ cm^3$ of liquid into a tube of diameter 3 mm., to what length would the tube be filled? Answer: $\approx$ 141 mm.

Solving for height in the formula $V = \pi r^2 h$, we have



$$1000\ mm^3 = 3.14 \times 1.5^2 \times h$$
$$1000 = 3.14 \times 2.25 \times h$$
$$1000 = 7.06\ h$$
$$141 \approx h$$

Exercise 5-8a

Using the data shown above and Figure 5-8, answer the following questions.

1. The length of tubing from the syringe to the bubble (drop of liquid) was 10 cm. What is the volume of gas (air) in the tubing (answer in cubic mm.)?

Answer: 10 cm. = 100 mm. Area of base was ≈ 7 mm. Thus, 700 mm^3 of air.

2. The volume of air in the syringe was 7 cc. Express this as mm^3.
   Answer: Volume of air in the syringe was 7 cc. = 7000 mm^3.

3. What is the total volume of gas at the time the experiment started?
   Answer: Total volume of air at the start of the experiment = 700 + 7000 = 7700 mm^3.

4. How much gas (in cubic mm) was produced during the first minute?
   Answer: Gas produced in one minute equals the difference between zero reading and the first minute or 16 mm. of distance (from graph).
   Using $V = \pi r^2 h$, then

   $V \approx 3.14 \times 1.5^2 \times 16$

   $V \approx 3.14 \times 2.25 \times 16 \approx 113\ mm^3$

5. How much gas was produced by the end of 2 minutes?
   Answer: Gas produced in 2 minutes = 36 mm. - 8 mm. = 28 mm.

   $V = \pi r^2 h$

   $V \approx 3.14 \times 2.25 \times 28$

   $V \approx 198\ mm^3$

6. At the end of the experiment, how much gas (in cubic mm.) has been produced?
   Answer: Total gas produced = 158 mm. - 8 mm. = 150 mm.

   $V = \pi r^2 h$

   $V \approx 3.14 \times 2.25 \times 150 \approx 1060\ mm^3$.

7. What is the total volume of gas in the system at the end of the experiment?
   Answer: Total volume of gas in system = original gas + produced gas = 7500 + 1060 = 8560 mm^3

8. The volume of liquid (expressed in cubic mm.) in our syringe was _____?
   Answer: Volume of liquid in syringe was 5 cc. = 5000 mm^3

9. What is the ratio of volume of liquid to gas produced in 10 minutes?
   Answer:

   $$\text{Ratio: } \frac{\text{volume of liquid}}{\text{gas produced}} = \frac{5000}{1060} \approx 4.7$$

10. If the liquid continued to produce gas at the same rate, in how many minutes would the volume of gas produced equal the volume of liquid?

Answer: A proportion would work well here: $\frac{5000}{1060} = \frac{x}{10}$

If 1060 mm^3 in 10 minutes, then 5000 mm^3 in "x" minutes. We have already found that $\frac{5000}{1060} = 4.7$; therefore, $4.7 = \frac{x}{10}$

$\therefore x \approx 47$ minutes.

11. Theoretically, how much gas would be produced in our system in 1 hour?
Answer: In one hour the gas produced would be 6 times as much as 10 minutes or $1060 \times 6 = 6360$ mm^3.

12. Gangbusters: At the same rate how much gas would be produced in one year (365 days)? Express your answer in scientific notation and in terms of cubic millimeters to two significant digits.
Answer: Gas produced in 365 days would be equal to gas produced in 1 hour × 24 hours × 365 days.
$6360 \times 24 \times 365 \approx 55{,}713 \approx 56$ million.
(Remind students of significant digits.)
Answer in scientific notation: 5.6×10^7 mm^3.

13. Express your answer to Problem 12 without scientific notation in cubic meters.
Answer: $1{,}000{,}000$ $mm^3 = 1$ m^3;
thus, $5.6 \times 10^7 = 56 \times 10^6$ and $10^6 = 1$ million;
therefore, 56 m^3.

14. Now, go back to Sections 5.2 and 5.7, first paragraphs, and review the information given there. Assuming that 20 per cent of the air in the syringe was oxygen, at what theoretical point in the generation of CO_2 by the yeast did respiration cease and fermentation begin?
Answer: Point of time when respiration ceased and fermentation began would be when the theoretical supply of oxygen was used up. If oxygen is 20 per cent of the volume of air, then 20 per cent of 7000 mm^3 = 1400 mm^3. A proportion would show that if 1060 mm^3 was produced in 10 minutes, then 1400 mm^3 was produced in x minutes.

$\frac{1060}{1400} = \frac{x}{10}$ $\qquad x \approx 13.2$ minutes

Exercise 5-8b

The students are asked to use their own equipment and data to answer these questions. The answers will vary but should be recognized as reasonable by the students.

1. - 7. Depend on student data.

8. Oxygen consumption should be approximately equivalent to carbon dioxide production.

Sample Test Items
Chapter 5

Matching

(d) 1. does not contain chlorophyl
(f) 2. without oxygen
(g) 3. data between two known amounts
(j) 4. volume of a rectangular prism
(b) 5. volume of a cylinder
(h) 6. data beyond a known amount
(c) 7. releases oxygen into the air
(a) 8. taking in oxygen and giving off carbon dioxide
(e) 9. with oxygen
(f) 10. 3560 mm^3

a) respiration
b) $\pi r^2 h$
c) photosynthesis
d) yeast
e) aerobically
f) anaerobically
g) interpolation
h) extrapolation

i) 3.56 cm^3
j) l w h

Use the data from the following table to answer Questions 11 through 15.

First term	Second term
5	100
15	151
25	203
35	256
45	310
55	365
65	431
75	488
85	546
95	605

11. What would be the second term for the ordered pair (15,151)?

12. What would be the first term for the ordered pair (75,488)?

13. Find the second term for a first term of 40. 283

14. Find the second term for the ordered pair (83,534.4)

15. Find the second term for the first term of 91. 581.4

A round bottle has the base area of 6.84 cm^2. What would be the volume if it were filled to the following heights:

Answer: (correct number of significant digits)

16. 2.0 cm. — 14 cm^3

17. 35 mm. — 24 cm^3 or 24,000 mm^3

18. 1.00 dm. — 68.4 cm^3 or .068 dm^3

19.	2.50 cm.	17.1 cm^3
20.	45.000 mm.	30.1 cm^3 or 30,100 mm^3

Multiple choice

		Answer
21.	How many square decimeters are there in 13 square meters? a. 130 b. 1300 c. 13,000 d. 130,000 e. none of these	b
22.	How many cubic decimeters are there in 13 cubic meters? a. 130 b. 1300 c. 13,000 d. 130,000 e. none of these	c
23.	What is the volume of a rectangular box 43 centimeters long, 3.0 decimeters wide, and 120 millimeters high?	$\approx$ 150 cm^3
24.	What is the volume of a cylinder 12 decimeters high with a diameter of 10 decimeters? ($\pi \approx 3.14$)	$\approx$ 940 dm^3

Chapter 6

GROWTH OF MOLD:
RECTANGULAR COORDINATES, ESTIMATION OF AREA

Teacher's Commentary

6.1 Introduction

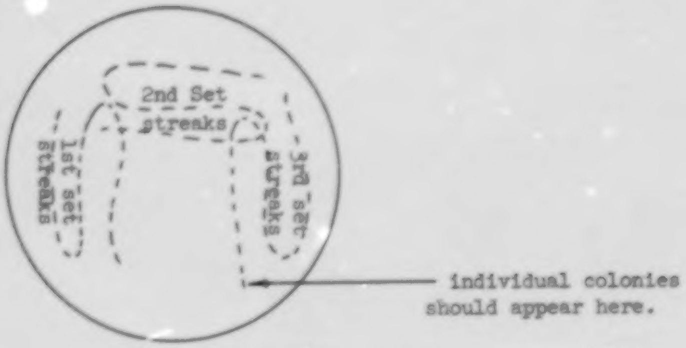
This third major biological concept which was briefly alluded to in Chapter 2 is here more fully developed, and the resulting graphs should be quite characteristic. If growth can be measured early enough and late enough in life of the organ (such as heart, liver, brain, etc.), organism (total living thing), or population, then three phases should be evident in the final graph:

(1) The lag phase, illustrating a slow start, a period of cell or organism adjustment.

(2) The grand phase or exponential phase, where the cells are multiplying exponentially (1 cell divides into 2, the 2 divide into 4--8--16--32--64--etc., becoming astronomical in number before limiting factors begin to be felt), - and

(3) The stationary phase or senescence, a leveling off as a result of a complicated set of limiting factors, such as metabolism, regulation, food supply, and relationships of the organisms with each other and with their environment.

Students might ask how one isolates a single bacterium, thereby starting a colony which acts precisely the same as any other population. A bacteriologist may have several techniques of isolating a single bacterium. One of these is by a method called "streaking." The sterile needle is dipped into a culture of bacteria, then drawn lightly across a petri dish containing sterile agar growth medium in one direction three or four times, then across these streaks perpendicularly several times, and again across the second set of streaks, thus scattering the bacteria so that colonies arising from an individual bacterium do result.



In reference to the story of rabbits in Australia, many "controls" were tried. At first, the people tried herding them together and bludgeoning them to death as is done in some plains states to this day. This, however, was found to be ineffective. Recently ecologists have started introducing natural enemies of the rabbit, such as the Red Fox. But it wasn't until the fungus disease Coccidio mycosis was introduced that the rabbits were brought under effective control.

Since there is little reliable data available on total world population, over a period of thousands of years, it is difficult to present this with any mathematical certainty. However, there have been many broad statements made. In the film Our Mr. Sun, produced by the Bell Telephone Company, the statement is made that "one out of twenty of all the people who ever lived in the world is alive today." It is from statements and estimates such as this, as well as data similar to that provided at the end of this chapter that the term "population explosion" is derived. The graph on population in the United States from 1660 to 1960 is meant to be a climactic ending to the chapter, so it is suggested the teacher does not refer the students to it at this point.

6.2 Coordinates

This section starts out with an informal introduction of identification of points in a plane. The idea to stress is that of a reference point and reference lines. For this set of exercises, our reference lines will be at right angles to each other (rectangular coordinates).

Exercise 6-2

Answers will depend upon your classroom seating arrangement.

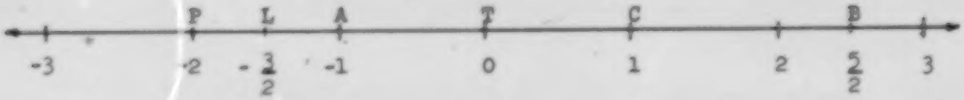
6.21 Coordinates on a Line

Identify the terminology of number, coordinate, distance, and direction. Rationals are used as the domain even though the students may be aware of placing irrationals on the number line.

Exercise 6-21

1. Draw a segment of a number line 6 inches in length. Mark off segments of length one inch and place the origin at its midpoint. On the line locate the following points:

   $A(-1)$, $B(\frac{5}{2})$, $C(1)$, $T(0)$, $L(-\frac{3}{2})$, $P(-2)$.



2. (a) In Problem 1, how far is it in inches between the point labeled T and the point labeled L ? $1\frac{1}{2}$ "

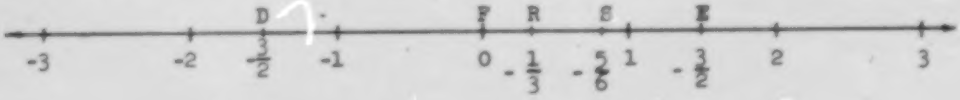
   (b) Between P and B ? $4\frac{1}{2}$ "

   (c) Between L and B ? 4 "

   (d) From the origin to A ? 1 "

3. Using a number line with 1 inch as the unit of length, mark the following points:

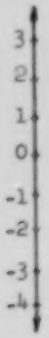
   $R(\frac{1}{3})$, $S(\frac{5}{6})$, $D(-\frac{3}{2})$, $F(0)$, $E(\frac{3}{2})$.



4. If the line segment in Problem 3 were a highway and if it were drawn to a scale of 1 inch representing 1 mile, how far in miles would it be between these points on the highway:

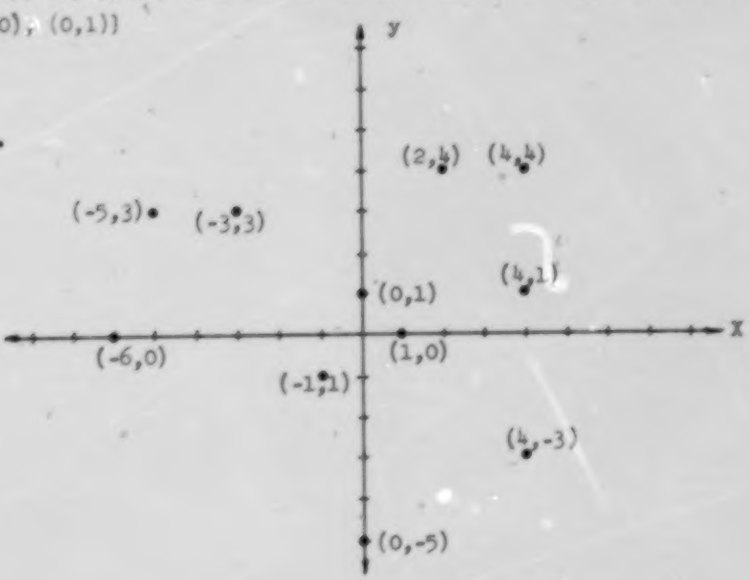
(a) F and R ? $\frac{1}{3}$ mile (b) D and E ? 3 miles

5. Draw a number line in a vertical instead of horizontal position. Mark your number scale with positive numbers above the origin and negative numbers below the origin. Label points to correspond with the rational numbers 0, 1, 2, 3, -1, -2, -3, -4.



Exercise 6-22

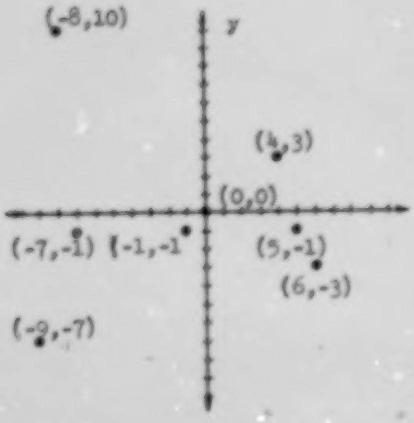
1. Given the following set of ordered pairs of rational numbers, locate the points in the plane associated with these pairs.
{(4,1), (1,0), (2,4), (4,4), (-1,-1), (-3,3), (4,-3), (-5,3), (0,-5), (-6,0), (0,1)}

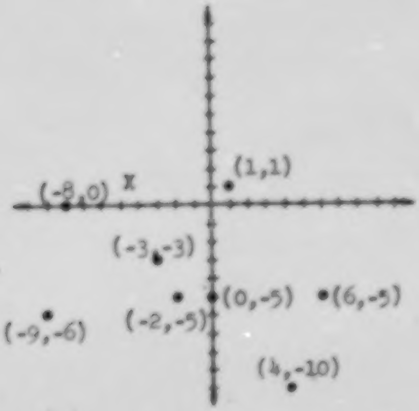


2. On squared paper draw a pair of axes and label them. Plot the points in the following sets. Label each point with its coordinates. Use a different pair of axes for each set.

Set A = {(6,-3), (-7,-1), (-9,-7), (5,-1), (-8, 10), (0,0), (-1,-1), (4,3)}

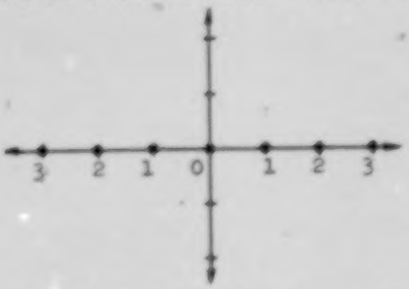
Set B = {(1,1), (6,-5), (-3,-3), (4,-10), (-9,-6), (-8,0), (0,-5), (-2,-5)}





3. (a) Plot the points in the following set:

Set C = {(0,0), (-1,0), (-2,0), (+2,0), (-3,0), (+3,0), (+1,0)}



(b) Do all of the points named in Set C seem to lie on the same line?

Yes

(c) What do you notice about the y-coordinate for each of the points?

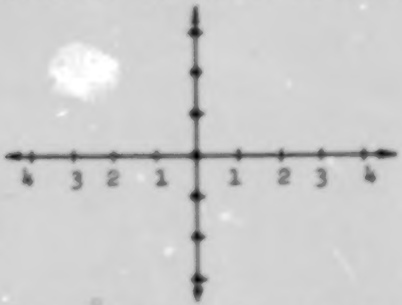
y = 0

(d) Are there any points on this line for which the y-coordinate is different from zero?

No

4. (a) Plot the points in the following set:

Set D = {(0,0), (0, -1), (0, 1), (0, 2), (0, -3), (0, 3), (0,-2)}



(b) Do all of the points named in Set D seem to lie on the same line?

Yes

(c) What do you notice about the x-coordinate for each of the points?

x = 0

(d) Are there any points on this line for which the x-coordinate is different from zero?

No

Exercise 6-23a

1. Given the following ordered pairs of numbers, write the number of the quadrant, if any, in which you find the point represented by each of these ordered pairs. (A number without a sign is understood to be a positive number.)

Ordered Pair	Quadrant
(a) (3,5)	I
(b) (1,-4)	IV
(c) (-4,4)	II
(d) (-3,-1)	III
(e) (8,6)	I
(f) (7, -1)	IV
(g) (-3,-5)	III
(h) (0,7)	none

2. (a) Both numbers of the ordered pair of coordinates are positive. The point is in Quadrant I .

(b) Both numbers of the ordered pair of coordinates are negative. The point is in Quadrant __III__ .

(c) The x-coordinate of an ordered pair is negative and the y-coordinate is positive. The point is in Quadrant __II__ .

(d) The x-coordinate of an ordered pair is positive and the y-coordinate is negative. The point is in Quadrant __IV__ .

3. (a) If the x-coordinate of an ordered pair __is__ zero and the y-coordinate __is not__ zero, where does the point lie?

   __On the y-axis, but not at the origin__.

   (b) If the x-coordinate of an ordered pair __is not__ zero and the y-coordinate __is__ zero, where does the point lie?

   __On the x-axis, but not at the origin__.

   (c) If __both__ coordinates of an ordered pair __are__ zero, where is the point located?

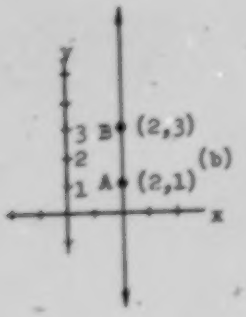
   __At the origin__.

4. Points on either the X-axis or the Y-axis do not lie in any of the four quadrants. Why not?

   __The quadrants are defined by the intersection of half-planes which do not contain the axes. Hence, their intersection does not contain the axes__.

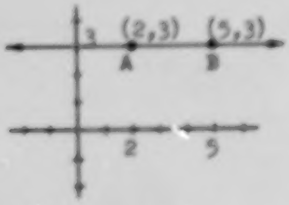
__Exercise 6-23b__

1. (a) Plot the points of set L = {A(2,1), B(2,3)}.
   (b) Use a straightedge to join A to B. Extend line segment AB.
   (c) Line AB seems to be parallel to which axis?



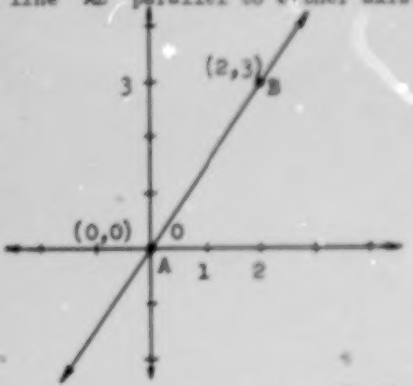
(c) the Y-axis

2. (a) Plot the points of set M = {A(2,3), B(5,3)}.
   (b) Use a straightedge to join A to B. Extend line segment AB.
   (c) Line AB seems to be parallel to which axis?



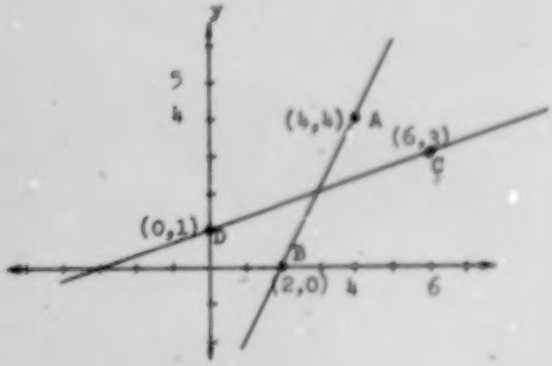
(c) the X-axis

3. (a) Plot the points of set N = {A(0,0), B(2,3)}.
   (b) Join A to B. Extend line segment AB.
   (c) Is line AB parallel to either axis?



(c) neither axis. AB is an oblique line.

4. (a) Plot the points of set P = {A(4,4), B(2,0)}.
   (b) Join A to B. Extend line segment AB.
   (c) Plot the points of set Q = {C(6,3), D(0,1)}.
   (d) Join C to D. Extend line segment CD.
   (e) What is the intersection set of lines AB and CD ?

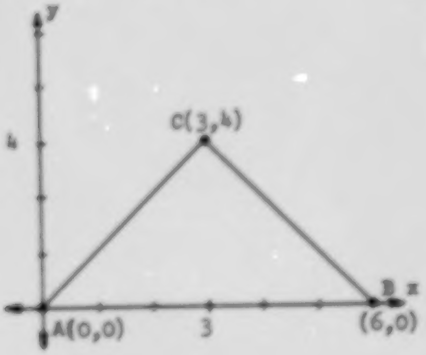


(e) {(3,2)}

5. (a) Plot the points of set R = {(A(0,0), B(6,0), C(3,4)} on the coordinate plane.

(b) Use a straightedge to join A to B, B to C, C to A.

(c) Is the triangle (1) scalene, (2) isoceles, or (3) equilateral?



(c) isoceles

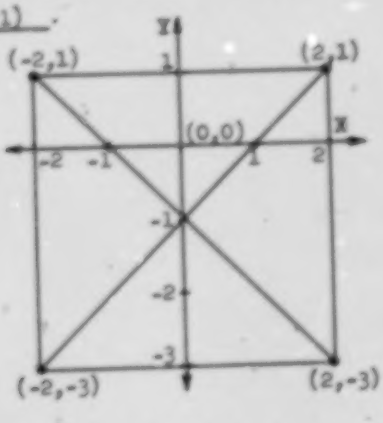
6. (a) Plot the points of set S = {A(2,1), B(-2,1), C(-2,-3), D(2,-3)}.

(b) Use a straightedge to join A to B, B to C, C to D, and D to A.

(c) Is the figure a square? Yes.

(d) Draw the diagonals of the figure.

(e) The coordinates of the point of intersection of the diagonals seem to be (0, -1).



7. (a) Plot the points of set T = {A(2,1), B(3,3), C(-2,3), D(-3,1)}.

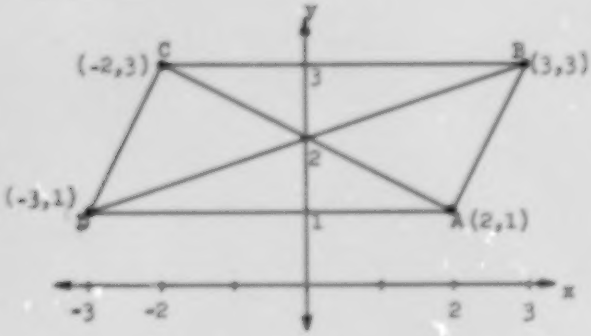
(b) Use a straightedge to join A to B, B to C, C to D, and D to A.

(c) What is the name of the quadrilateral formed?

parallelogram

(d) Draw the diagonals of the quadrilateral ABCD.

(e) The coordinates of the point of intersection of the diagonals seem to be (0,2) .



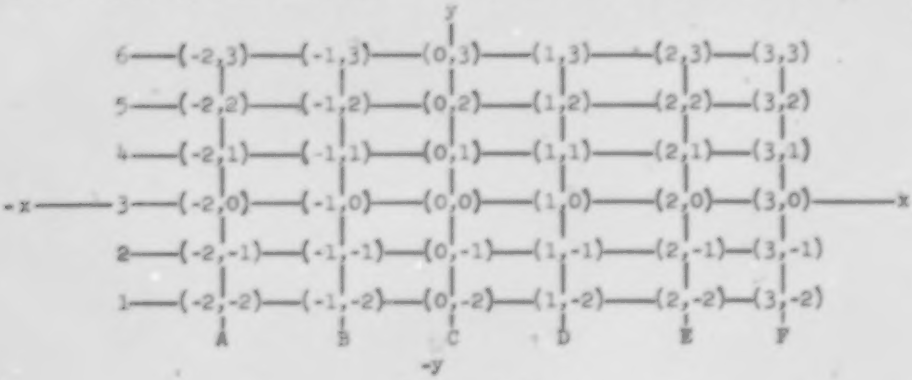
6.24 Additional Activity with Coordinates - "Living Coordinates"

The following demonstration can be used successfully to enhance and reinforce the understanding of the coordinate system.

A preliminary introduction should be given first. Section 6-2 in the student text would give such an introduction.

Living Coordinates involves every pupil in the entire class in every problem. Each pupil represents a point on a plane, identified by an ordered pair. He must be both attentive and knowledgeable to "stay alive" during this exercise.

Arrange the student chairs in rows and columns. Preferably in an uneven number of rows and columns, but since most math classes have 36-40 students, an even number of rows or columns is acceptable. See Figure 6-2a.



	A	B	C	D	E	F
6	(-2,3)	(-1,3)	(0,3)	(1,3)	(2,3)	(3,3)
5	(-2,2)	(-1,2)	(0,2)	(1,2)	(2,2)	(3,2)
4	(-2,1)	(-1,1)	(0,1)	(1,1)	(2,1)	(3,1)
3	(-2,0)	(-1,0)	(0,0)	(1,0)	(2,0)	(3,0)
2	(-2,-1)	(-1,-1)	(0,-1)	(1,-1)	(2,-1)	(3,-1)
1	(-2,-2)	(-1,-2)	(0,-2)	(1,-2)	(2,-2)	(3,-2)

Figure 6-2a

Explain that the pupils are a living coordinate system. Draw a diagram similar to the seating chart on the chalk board. Mark the coordinates of each seat (point) on the chalk board. Be sure each student knows what ordered pair he represents.

Rows and columns are written on the diagram for purposes of identifying the correct answer to you. Do not refer to them or write them on the chalk board. If an answer needs to be identified to the class, point to the diagram on the chalk board. The students should each have their own identifying ordered pair written on a piece of paper and on their desk. They will need to refer to it frequently.

The demonstration consists solely in students standing as a response to specific questions. The following are some sample questions of varying degrees of difficulty (not necessarily in order of degree of difficulty). We are sure the teacher will think of many more and better questions.

Sample Questions

1. (a) All of the students in the first quadrant stand.
   The sample coordinate system in Figure 6-2a shows these would be (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), not those on the axes.
   (b) All of the students in the second quadrant stand.
   (c) All of the students in the third quadrant stand.
   (d) All of the students in the fourth quadrant stand.

2. (a) The "Y"-axis stand.
   These would be from (0,-2) to (0,3) including the origin (0,0).
   (b) The "X"-axis stand.
   These would be from (-2,0) to (3,0) including the origin (0,0).

3. (a) Second and third quadrants stand.
   Be sure those on the "X"-axis do not stand (Column 3).
   (b) Third and fourth quadrants stand.
   Watch out for those on the "Y"-axis (Row C).
   (c) First and fourth quadrants stand.
   (d) First and second quadrants stand.

4. Those whose abscissas and ordinates are equal. The abscissa is the first term of an ordered pair; the ordinate is the second term.
A sloping line through the origin from (-2,-2) to (3,3).
Be sure the "origin" stands.

5. All whose abscissas are 1 stand.
Straight line, Row D from (1,-2) to (1,3).

6. Those whose abscissa is -1 stand.
Row B from (-1,-2) to (-1,3).

7. Those whose abscissa is 0 stand.
The "Y"-axis.

8. Those with an ordinate of 2 stand.
Line parallel to the "X"-axis, Column 5, from (-2,2) to (3,2).

9. All those with an ordinate of 3 stand.
A straight line parallel to the "X"-axis, (-2,3) to (3,3).

10. Those with a negative abscissa stand.
All those left of the "Y"-axis, including part of the "X"-axis.

11. Those with a positive ordinate stand.
All "above" the "X"-axis. Zero is neither positive nor negative, so the "X"-axis does not stand.

12. All those whose ordinate is not negative stand.
All those "above" the "X"-axis and the "X"-axis, as zero is not negative.

13. Those whose abscissa and ordinate have opposite signs stand.
Second and fourth quadrants.

14. (a) All with an abscissa of 3 stand and remain standing.
    (b) All with an ordinate of -2 stand and remain standing.
    (c) Those who meet both of the previous conditions be seated.
    Only (3,-2) should sit down.

15. Those with an ordinate of +2 or -2 stand.
Two lines parallel to the "X"-axis (Columns 1 and 5).

16. All whose sum of abscissa and ordinate is zero stand and remain standing through instruction 18.

From (-2,2) through origin to (2,-2).

17. All whose sum of abscissa and ordinate is a positive number stand and remain standing through instruction 18.

A little hard to check; will include some on the axes and some in quadrants I, II, IV.

18. All whose sum of abscissa and ordinate is a negative number stand.

The entire class should be up after 16, 17, and 18.

Ladies and gentlemen - be seated.

19. Those students whose ordinate is less than 2 stand.

The front four columns should stand, last two columns have ordinates greater than or equal to ($\geq$) 2.

20. Everyone whose abscissa is greater than 3 stand.

No one should stand.

21. All whose sum of abscissa and ordinate is less than seven stand.

All should stand.

After an introduction to linear equations and inequalities (probably first year algebra) many more living solutions may be graphed: Intersections of lines, and mathematical sentences which require whole regions or the intersection of regions to respond. "And, or" situations may be called for also.

6.3 Activity - Growing Mold

Materials and Supplies

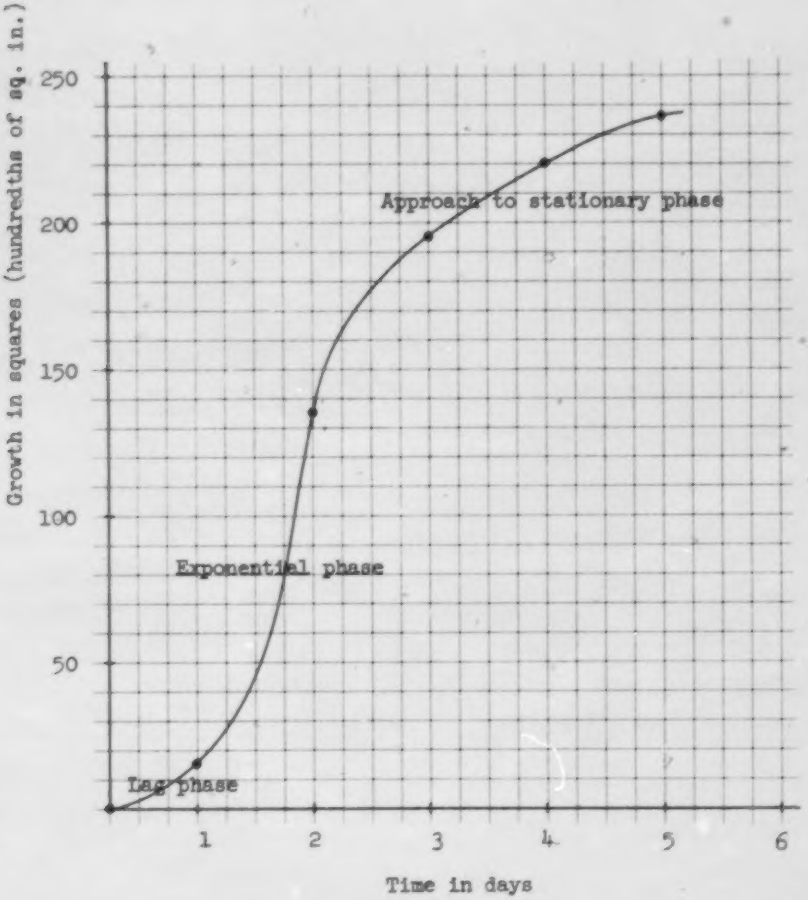
Aluminum pie or cake tin: approximately nine-inch diameter. One per team. Students may be asked to bring one from home, provided it does not have to be returned. At the end of the exercise these should be disposed of. Another source would be from a variety store.

Recording

1 and 2. The suggestion given in the student text for estimating the growth of mold is based on the graph paper having small space units. If "10 × 10 to the inch" paper is used, then each small square will equal one - one hundredth of a square inch.

3. The data table form is only a suggestion. Students may come up with a form that they like better.

The following graph shows the results obtained during five days of growth in an activity carried out according to the instructions in the text.



Graph paper: Eight sheets per student plus two extra sheets per team. The instructions in the student text are for 10 × 10 to the inch graph paper. If another kind is used instructions will have to be altered to fit.

Gelatin: Colorless if possible, such as Knox. One package per 10 - 12 teams.

Bouillon cubes: One cube per 10 - 12 teams.

Saran Wrap: One roll.

Rubber bands: One per team (large enough to fit snugly around tin). Marking or masking tape will also hold Saran wrap.

Procedure

1. Marking and cutting the graph paper to be placed in the bottom of the tin should not be troublesome if directions in student text are followed.
2. Preparation of gelatin-bouillon mix could be done by one student for the entire class. Notice instruction in student text for cold water with gelatin and hot water with bouillon. The only anticipated problem might arise when the mix is prepared too far in advance. It sets rapidly. A very thin layer of mix in each tin is sufficient. Just enough to moisten the graph paper - pour excess back into container. When the layer is too thick, reading of squares on the graph paper becomes difficult.
3. The word "contaminate" is used instead of the term "inoculate" because of the connotation associated with the latter. The growth medium is not being inoculated with mold spores by hand and at specific points on the medium, but rather is being contaminated or polluted by random falling of spores from the air.
4. Mold growth will be most pronounced when the tins are stored in a dark area and where the temperature is fairly uniform and warm.
5. It is recommended that the tins be prepared on Friday so that mold growth will be apparent on Monday. This will give five successive days for readings.

Questions

1. At completion of seven days of recording and graphing, were your predictions (extrapolations) in all cases correct?

   Probably not - unless students are aware that data of this kind when graphed will not result in a linear graph.

2. If not, at what points did they fail?

   It is doubtful that the student would predict either the exponential phase or the stationary phase.

3. What tentative explanations can you give for any deviations from a linear graph?

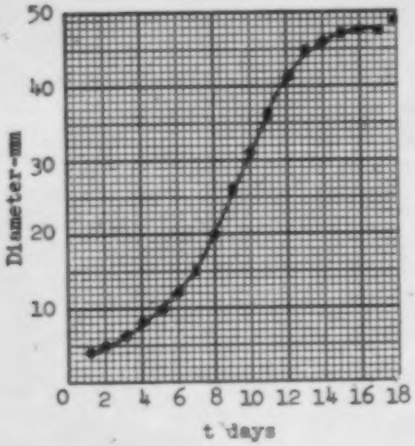
   These explanations are in the introduction to this chapter.

Exercises 6-3

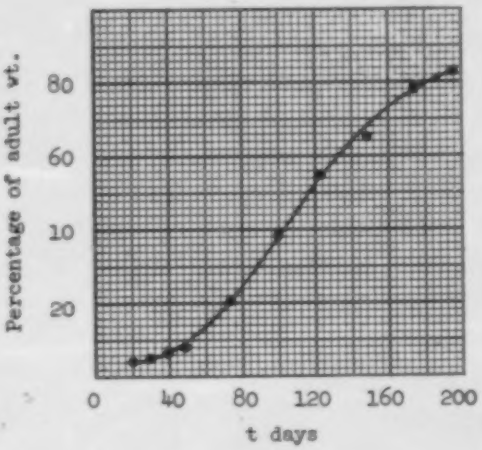
These graphs illustrate the typical "Sigmoid" ("S") curve of growth. In the case of organs and organisms, the stationary phase is a result of metabolic factors usually controlled by the inherent (inherited) genetic make-up of the organism involved. In populations, the limits are imposed by environmental factors - space, food available, accumulating toxic wastes, etc.

Answers:

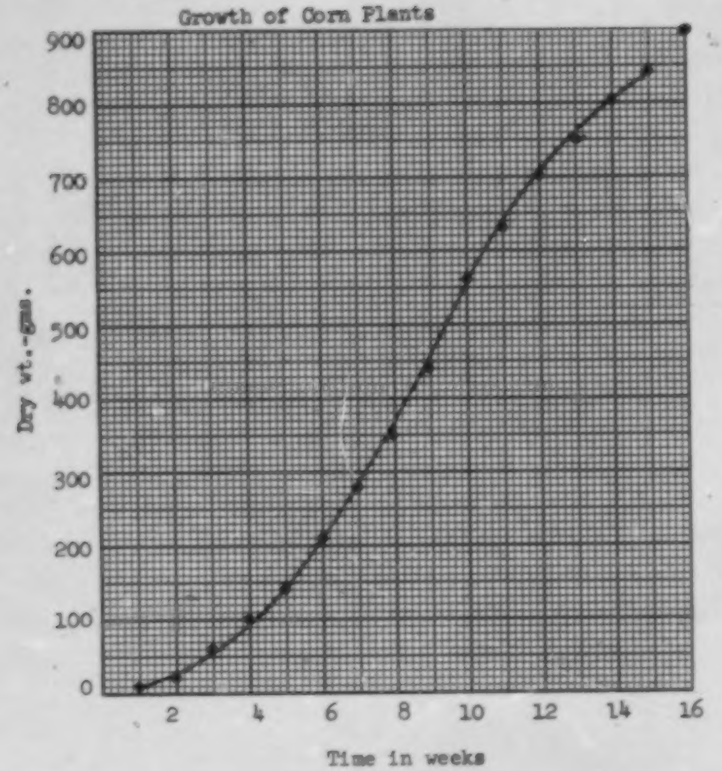
1. Growth of Gourd Fruit



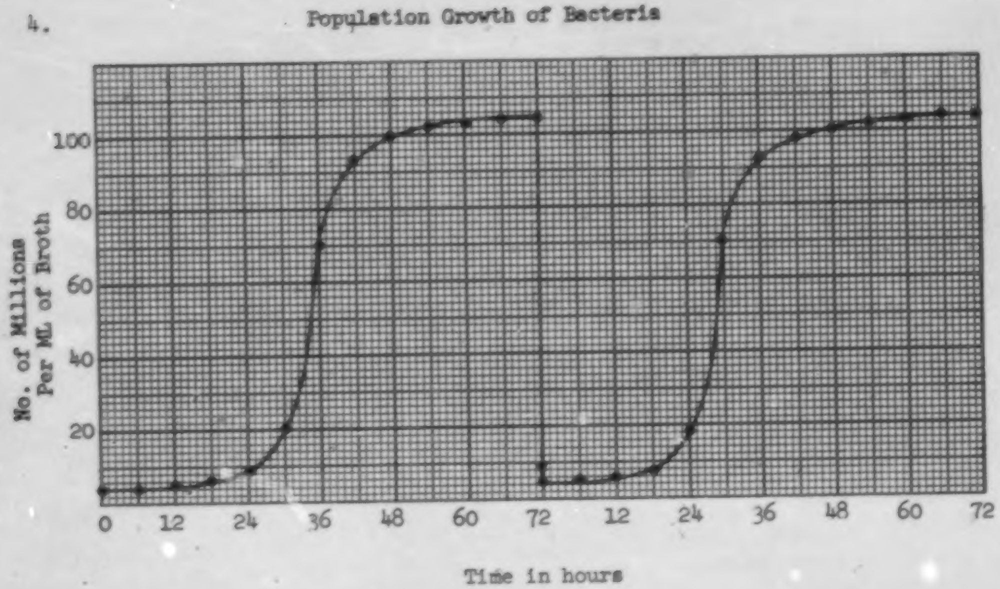
2. Growth of Chick



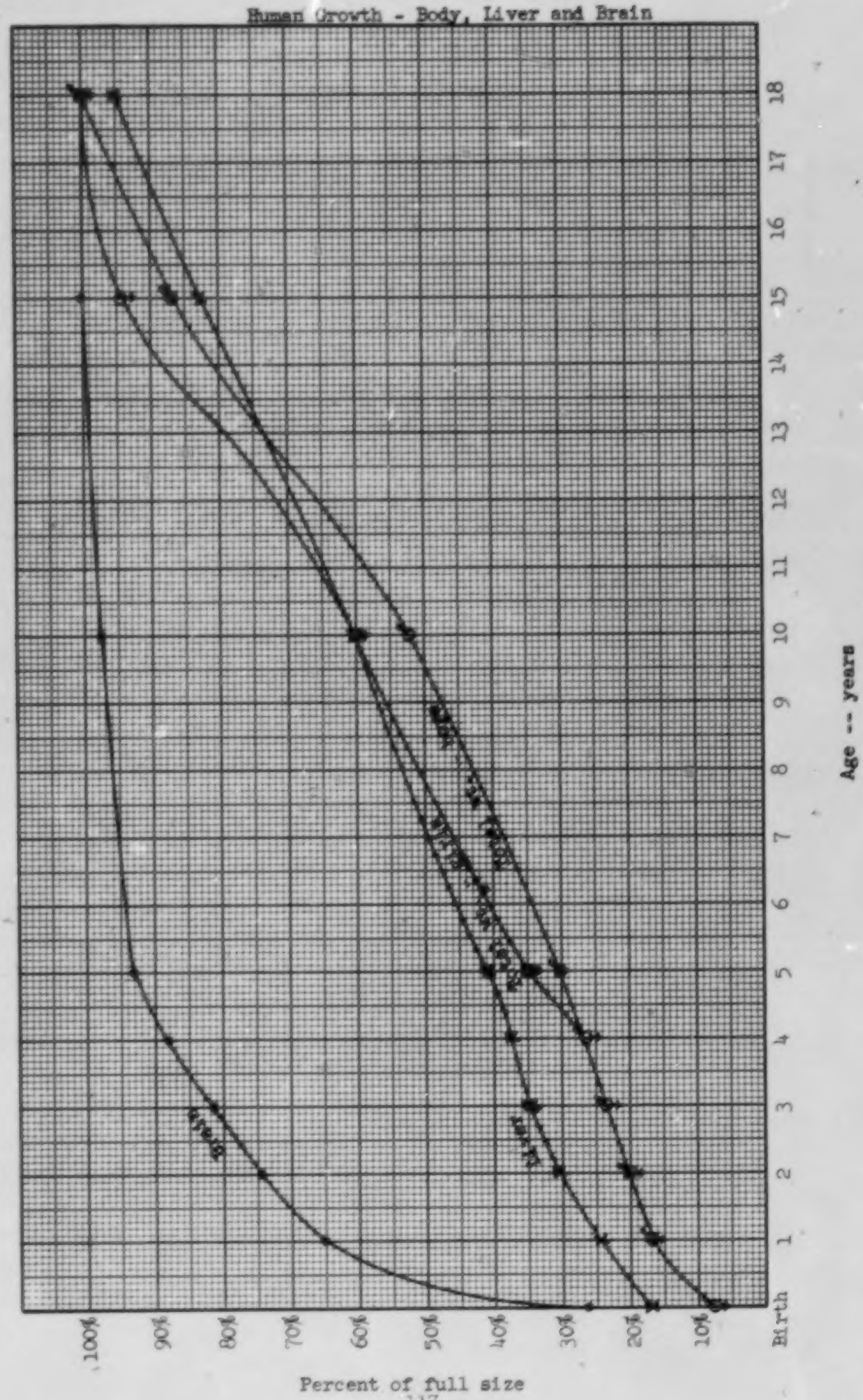
3.


Growth of Corn Plants
900
800
700
600
500
400
300
200
100
0
Dry wt.-gms.
2
4
6
8
10
12
14
16
Time in weeks

4.

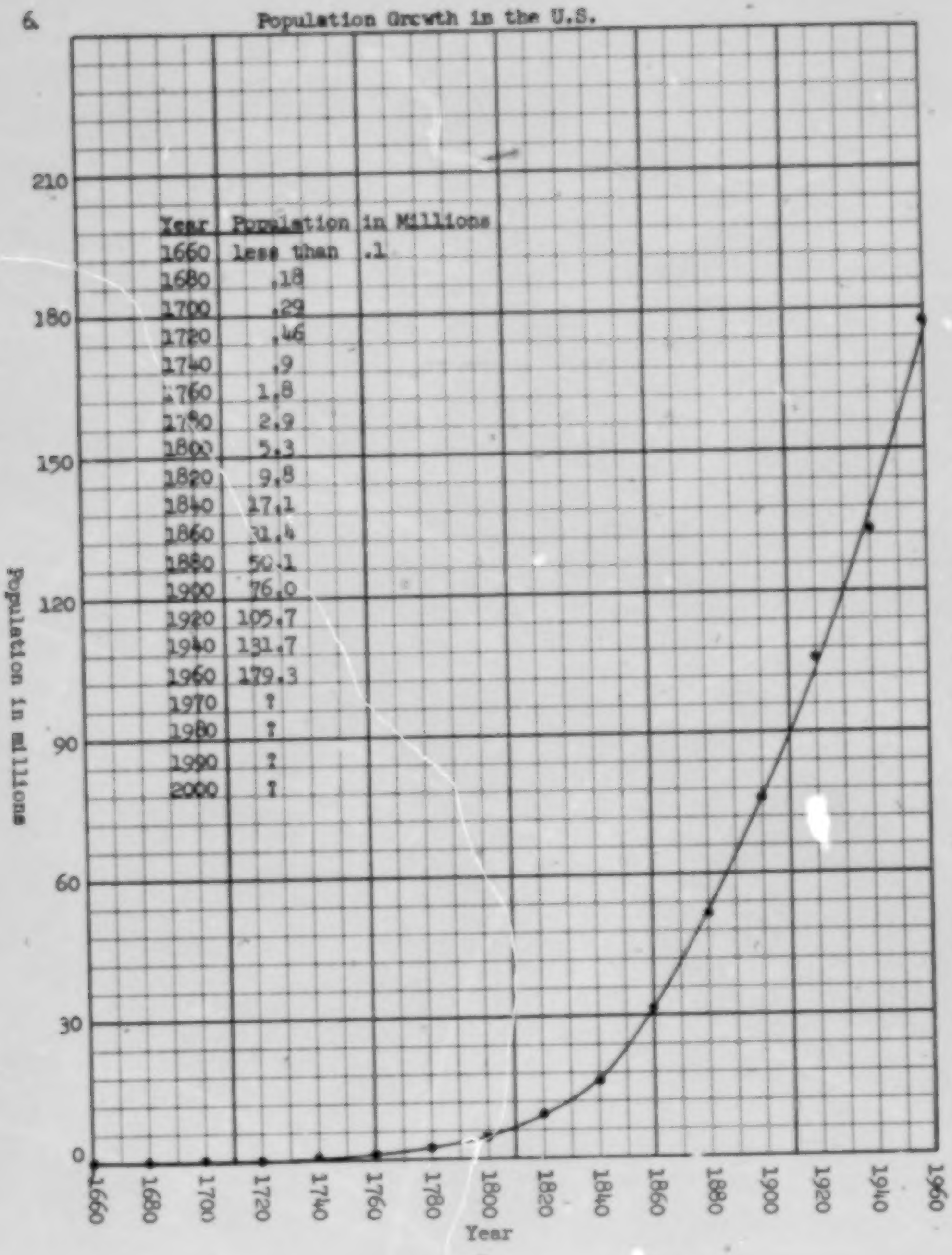

Population Growth of Bacteria
100
80
60
40
20
No. of Millions Per ML of Broth
0
12
24
36
48
60
72
12
24
36
48
60
72
Time in hours

5. Human Growth - Body, Liver and Brain



6.

Population Growth in the U.S.



Year	Population in Millions
1660	less than .1
1680	.18
1700	.29
1720	.46
1740	.9
1760	1.8
1780	2.9
1800	5.3
1820	9.8
1840	17.1
1860	31.4
1880	50.1
1900	76.0
1920	105.7
1940	131.7
1960	179.3
1970	?
1980	?
1990	?
2000	?

Sample Test Items

1. What is one mathematical name for (2,3)?	Answer: ordered pair or set
2. Do (2,3) and (3,2) represent the same point in a plane?	no
3. Would (3,1) and (6,2) represent the same point on a plane?	no
4. What is the name of the point which the axes have in common?	origin
5. In which quadrant is the point with coordinates (9,7)?	I
6. In which quadrant is the point with coordinates (5,-4)?	IV
7. Between which quadrants is the point with coordinates (-4,0)?	II and III
8. On the number line how many units apart are 14 and 9?	5
9. On the number line how many units apart are 11 and -7?	18
10. If a number line is horizontal, is 13 to the left or right of 17?	left

The following problems require coordinate paper. Would suggest that the test copy have the coordinates on it.

11. On the axis prepared, plot and record the following points with both their letter and coordination (be neat).
A(10,10) B(10,20) C(-5,10)
D(-7,-7) E(5,-5)

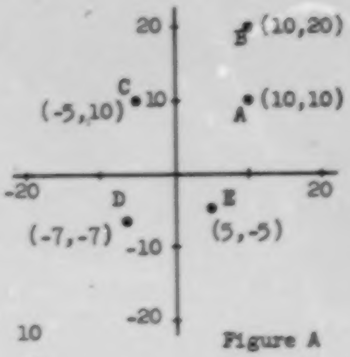
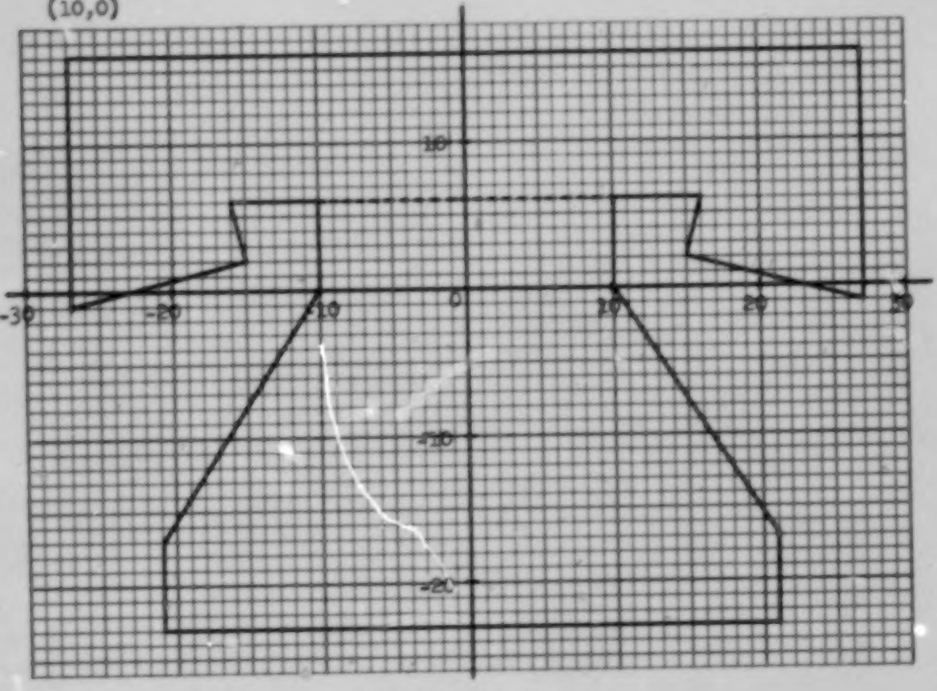


Figure A

In Figure A,

12. How many units long is segment AB?	10
13. How many units long is segment AC?	15
14. Which segment connecting any 2 of our 5 points would pass through the origin?	$\overline{AD}$
15. Which quadrant has the most points marked?	I

16. Plot the following sets of ordered pairs. After you have plotted each point, connect it to the point you plotted last with a line segment: (10,6), (16,6), (15,2), (27,-1), (27,16), (-27,16), (-27,-1), (-15,2), (-16,6), (-10,6), (-10,0), (-21,-17), (-21,-23), (21,-23), (21,-17), (10,0)



6.4 Second Activity - Growth of Crystals

Some teachers may have used this same type of solution in their youth on coal or cinders. The idea is not new. The authors can remember seeing it years ago. Several chemists, who were friends of the authors, were contacted to determine the safety of its use. They concluded that the solution used in the following sections is a chemical mixture and could be toxic if taken internally. The growth which develops is a very complex substance believed to be made up of very small crystals. This too could be toxic and care should be taken and students cautioned that this material should not come in contact with the mouth.

The tolerance in the recipe appears to be considerable. No great care need be taken in measuring. Bluing appears to be the "unknown" ingredient. Our bluing bottle did not list the chemicals it contained. According to some sources bluing is made from "Prussian Blue."

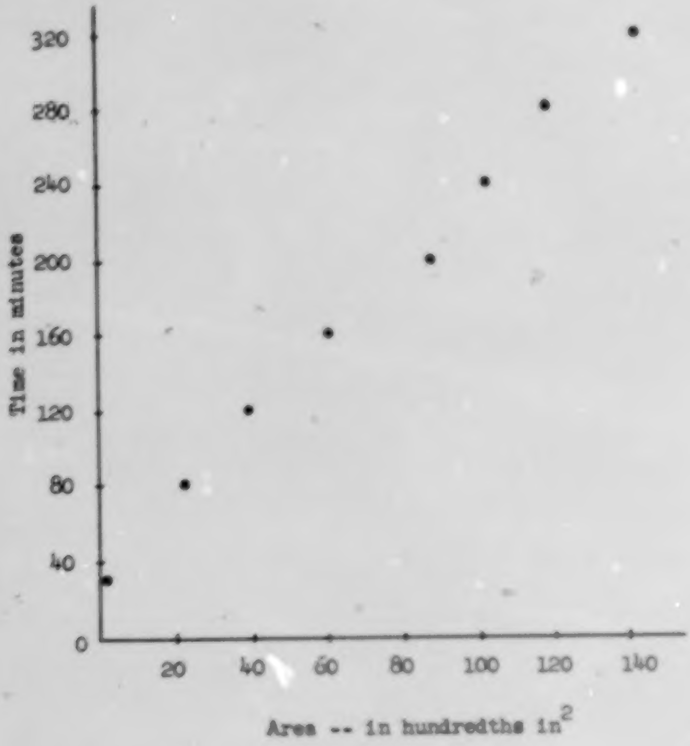
The growth will also appear through the edges of the milk cartons. Therefore all production should be over paper covered counters or plastic topped counters. Pie tins serve well as protective containers of the "overflow."

The authors are sure that student ingenuity will discover many alternate uses of the solution. Any porous substance seems to work. Even plastic covered window screen "wicked" the solution up.

Egg shells are porous. If an egg is "blown" (insides removed) and filled with the solution an interesting growth may occur.

The entire activity came about for a need to show rapid growth. This is of course not living but it is a growth. Hopefully the student will devise other methods of production which can be measured.

The following graph shows an area - time relationship of a growth as described in 6.4.



You will notice that very little mathematics is included in this section. One reason - no easily measurable method of production was devised other than the one prepared in 6.4. Most math rooms do not have devices for weighing or analyzing materials. One good justification for the activity - tremendous interest in the hourly and daily growth was expressed by other SMSG writers. They visited the MALT office frequently to "see what was going on." If students want to "get to their math classroom," just to see what has happened - if some new enthusiasm for the math period can be generated - then the activity has justified its existence.

You may decide to have various groups working on the different activities. As it is a growth do not expect any two experiments to come out the same.

Incandescent lamps will furnish heat to aid in the evaporation. A fan would also aid evaporation but the air currents would probably disturb the growth.

Chapter 7

SIZE OF CELLS AND METABOLISM:
SURFACE AREA AND VOLUME

Teacher's Commentary

7.1 Introduction

Through the medium of the relationship between surface area and volume, the threads of metabolism, unity, diversity and adaptation are here continued. It may be important for the teacher at this point to emphasize a situation which has not yet been mentioned. It should be obvious that the biological information which has been brought out so far has been presented by necessity with very little depth. The major biological threads which have been woven through the chapters, natural variation, adaptation and metabolism, touch lightly upon three of the major areas of research in biological sciences today.

Biological science has been said to be entering a period of explosive development and breakthroughs comparable to that of atomic physics during the first half of the century. Progress in molecular biology during the past few years has been electric. DNA (deoxyribo-nucleic acid), RNA (ribo-nucleic acid), and the hereditary code; ATP (adenosine-triphosphate), and energy (for cell division, protein synthesis, etc.); ecology (biological community relationships, population studies, and world biome concepts); and genetics (again DNA, "the code of life" and population genetics) are areas under intensive research. They are all so closely interwoven that it is impossible to actually separate them into categories as indicated above. They all hold so much promise for man: elimination of disease, increase of life span, maintenance of vigor into advanced years, creation of some simple forms of living things, replacement of defective genes with sound ones, artificial photosynthesis, or even acceleration of man's evolution.

The excitement of being on this threshold of dramatic and "earth-shaking" events (even more important to man than the advent of atomic energy) has lured many chemists and physicists into the field of biological research. Discussions of these topics have become more and more common in popular magazines such as Life and Reader's Digest. Paperback books on the subject are becoming more and more plentiful at book stores and newsstands.

Since molecules peculiar and vital to life are throughout the cell and are in a constant state of "flux," that is, the whole cell is dynamic and constantly changing, new materials must reach them constantly. The relationship between surface area and volume, then, becomes extremely important.. If a cell becomes larger than optimum, its metabolism slows, but usually the cell divides into two before this state is reached. An elephant's cells are the same size as those of a mouse. He just has more of them. If the students should bring up the ostrich egg (or any other egg), which is one cell--the largest known in the world today--it is important to point out to them that only an extremely minute portion of the egg is alive. The rest is stored food, held there for the developing embryo. Incidentally--the shell is porous--to allow exchange of gases. A chick embryo will die if the shell is coated with vaseline.

7.2 Activity - Construction of Solids

A regular polyhedron: the cube

The surface area - volume relationship of various sized cubes will be used later in the chapter. It is suggested that various sizes be made (i.e., two or three students each make one size, two or three each make another size, etc.) so that comparative relationships and differences can be seen when a master table is used for all data. Recommended measure for starting squares of paper would be 22 in., 20 in., 18 in., 16 in., ... 4 in. Paper squares larger than 8 inches per side can be cut from newspaper. A full double sheet is needed for the larger squares. It is not anticipated that students will have difficulty in actual folding of models. Students are asked to find a relationship between the measure of an edge of the cube and a measure of an edge of the starting square of paper. They should discover that this relationship is 1 to 4, the cube edge being $\frac{1}{4}$ of the edge of the square.

Exercise 7-2

Comparing Measures of a Cube

This exercise is included at this time to help students gain an appreciation of the fact that the rate of decrease of volume is considerably greater than the rate of decrease of surface area as the size of the cube is decreased. This relationship is finally to be compared to cell size and shape.

Complete the following table by filling in calculated values for the question marks.

Table 7-2

Surface Area and Volume of a Cube

Edge of Cube (s)	Surface Area in square inches	Volume in cubic inches	Ratio - S.A./Volume
12"	864	1728	1 : 2
9"	486	729	1 : 1.5
6"	216	216	1 : 1
3"	54	27	1 : .5
2"	24	8	3 : 1
1"	6	1	6 : 1

Following the construction of cubes, the students are given an opportunity to construct other polyhedrons to study the relationship of area and volume as these figures approach the shape of a sphere.

7.21 Construction of Regular Polyhedrons

The five known regular polyhedrons are included in this chapter for several reasons:

1. Historical interest.
2. Relative ease of constructing the "nets" (floor plan of the solids, Fig. 7-21, b and c).
3. Mathematical applications of "net" construction, the use of protractor and ruler in construction of pentagons and equilateral triangles.
4. Interest of building some geometrical figures.
5. Comparisons of surface area and volume.

The regular solids may be constructed with almost any length of edge. To compare them against each other, however, requires either volume or surface area to be constant for all of them. If the following dimensions are used as edge measures, the volumes are relatively constant and the surface area decreases as the number of faces increases. For the suggested dimensions, the volumes vary less than $2\frac{1}{2}$ percent. You may use a constant factor and increase all of the edge measures by $1\frac{1}{2}$, 2 or 3, etc.

As the comparison is between surface area and volume, one proper comparison would be to use a given volume for all and then compare the needed surface area to encompass it. By substituting 1000 cm^3 in each volume formula, we can find the measure of the edge for each solid. Using the measure of the edge computed in the surface area formula we find the total surface area. The following table does that (maintaining only 3 significant digits).

Volume	Surface Area
Tetrahedron	
Volume of Tetra $\approx .118s^3$	if $s \approx 20.4$ cm then
$1000\ cm^3 \approx .118s^3$	S.A. $\approx 1.732 \times 20.4^2$
$8474 \approx s^3$	$\approx 1.732 \times 41.61$
$20.4\ cm \approx s$	$\approx \underline{721}\ cm^2$
Hexahedron	
Volume of Hex $= 1.00\ s^3$	if $s = 10.0$ cm then
$1000\ cm^3 = 1.00\ s^3$	S.A. $= 6.000 \cdot 10.0^2$
$10.0\ cm = s$	$= \underline{600}\ cm^2$
Octahedron	
Volume of Octa $\approx .471s^3$	if $s \approx 12.8$ cm then
$1000\ cm^3 \approx .471s^3$	S.A. $\approx 3.464 \cdot 12.8^2$
$2123 \approx s^3$	$\approx \underline{568}\ cm^2$
$12.8\ cm \approx s$	
Dodecahedron	
Volume of Dodeca $\approx 7.66\ s^3$	if $s \approx 5.06$ cm then
$1000\ cm^3 \approx 7.66\ s^3$	S.A. $\approx 20.646 \cdot 5.06^2$
$130 \approx s^3$	$\approx \underline{529}\ cm^2$
$5.06\ cm \approx s$	
Icosahedron	
Volume of Icosa $\approx 2.18\ s^3$	if $s \approx 7.71$ cm then
$1000\ cm^3 \approx 2.18\ s^3$	S.A. $\approx 8.660 \cdot 7.71^2$
$459 \approx s^3$	$\approx \underline{515}\ cm^2$
$7.71 \approx s$	

Table 7-21

When Volume Remains Relatively Constant

(taken from above calculations)

Polyhedron	Measure of edge × 3 =	Total Resulting Measure of Edge	Resulting S.A. mm^2	Resulting Volume (constant) mm^3
Tetrahedron	20.4 mm	61 mm	6,444	26,784
Hexahedron	10 mm	30 mm	5,400	27,000
Octahedron	12.8 mm	38 mm	5,002	25,845
Dodecahedron	5.1 mm	15 mm	4,645	25,862
Icosahedron	7.7 mm	23 mm	4,581	26,548

Note decreasing S.A as volume remains relatively constant.

7.3 Surface Area Formulas for Regular Solids

Answers to Table 7-3a: Surface Area if edge measures 2 inches.

Table 7-3a

Regular Polyhedrons

Polyhedron	Number of Faces	Number of Edges	Number of Vertices	Shape of Face	Surface Area when Edge Measures 2 in.
Tetrahedron	4	6	4	triangle	6.928 sq. in.
Hexahedron	6	12	8	square	24.000 sq. in.
Octahedron	8	12	6	triangle	13.856 sq. in.
Dodecahedron	12	30	20	pentagon	82.584 sq. in.
Icosahedron	20	30	12	triangle	34.64 sq. in.

The objective here is to develop the formulas for areas of these figures when the measure of the edge is the only data given.

The use of compass, ruler, and protractor is needed in this section. Some review of their use may be in order.

Each student should do the constructions of equilateral triangles, pentagons, etc.

Some of the formula development may seem to be too involved for eighth graders. It could be *if* pushed through. After each student has read through the material, *slowly develop* the formula on the chalkboard. *Discuss each* step. Give reasonable *justification* for each step.

7.4 Pythagorean Property

The Pythagorean Property is introduced here in order to find the height of the equilateral triangular faces. The property is *not* proven as a theorem. It is merely recognized and used.

Many students are already familiar with the property and if the entire class is comfortable with it, the teacher may wish to move on rapidly. If not familiar, suggest a student project--have each construct squares of tagboard 3" square, 4" square and 5" square. When placed together they form the familiar Pythagorean triangle.

Exercise 7-4a

1. Show for each set that the square of the first number is equal to the sum of the squares of the other two numbers, for example, in the set of numbers 10, 8, and 6.

$$10^2 = 100 = 8^2 + 6^2 = 64 + 36 = 100,$$

thus,

$$10^2 = 8^2 + 6^2.$$

(a) 5, 4, 3

(b) 13, 12, 5

(c) 25, 7, 24

(d) 20, 16, 12

Answers:

(a) $5^2 = 4^2 + 3^2$
$25 = 16 + 9$
$25 = 25$

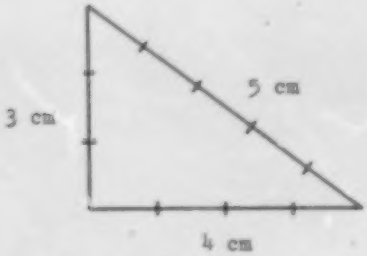
(b) $13^2 = 12^2 + 5^2$
$169 = 144 + 25$
$169 = 169$

(c) $25^2 = 7^2 + 24^2$
$625 = 49 + 576$
$625 = 625$

(d) $20^2 = 16^2 + 12^2$
$400 = 256 + 144$
$400 = 400$

2. Make a drawing of the triangle with the sides of length given in part (a) of Problem 1. Use your protractor to show that this triangle is a right triangle. (Use cm as the units.)

Answer:



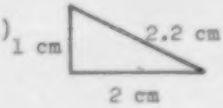
3. Draw right triangles, the lengths of whose shorter sides (in centimeters) are:

(a) 1 and 2 (b) 4 and 5 (c) 2 and 3

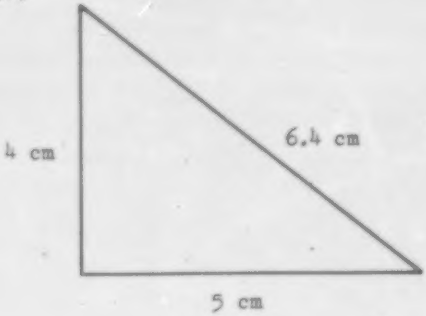
Measure, to the nearest one-tenth of a centimeter if possible, the lengths of the hypotenuses of these triangles.

Answer:

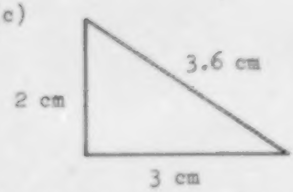
(a)



(b)



(c)



4. Use the Pythagorean Property to find the area of the square on the hypotenuse for each triangle in Problem 3.

Answers:

(a) 5 cm^2
$1^2 + 2^2 = 5$

(b) 41 cm^2
$4^2 + 5^2 = 41$

(c) 13 cm^2
$2^2 + 3^2 = 13$

The proof of the theorem by Pythagoras allows us to use it several ways. It is sometimes difficult for an eighth grader to see the difference between a theorem and its converse. Some simple illustrations will demonstrate that not all theorems work "both ways."

1. All horses are animals. Are all animals horses?
2. All fish have scales. Do all scales have fish (even the scales in your grocery store)?

Go over the three statements pertaining to the property with the students. The "If-then" should be emphasized.

This approach to square root is based upon an iteration system. Many text books use this method entirely. If you desire, this is also a logical place for teaching square root by the traditional algorithm. By iteration, to find the square root of a number, the student alternates division with averaging.

To find the square root of 20:

(1) Divide 20 by the closest guessed divisor.

$$\begin{array}{rr} & 5 \\ 4 & \overline{)\,20} \\ & \underline{20} \\ & 0 \end{array}$$

(2) Average divisor and quotient.

$4 + 5 = 9$. Average $= 4.5$

(3) Divide 20 by new divisor.

$$\begin{array}{rr} & 4.4 \\ 4.5 & \overline{)\,20.0} \\ & \underline{18\ 0} \\ & 2\ 00 \\ & \underline{1\ 80} \end{array}$$

(4) Average divisor and quotient.

$$\begin{array}{rr} & 4.5 \\ & \underline{4.4} \\ 2 & \overline{)\,8.9} \\ & 4.45 \end{array}$$

(5) Divide 20 by new divisor.

$$\begin{array}{rr} & 4.49 \\ 4.45 & \overline{)\,20.0000} \\ & \underline{17\ 80} \\ & 2\ 200 \\ & \underline{1\ 780} \\ & 4200 \end{array}$$

(6) Average:

$$\begin{array}{rr} & 4.45 \\ & \underline{4.49} \\ 2 & \overline{)\,8.94} \\ & 4.47 \end{array}$$

(7) Repeat until the desired number of significant digits is obtained.

Exercise 7-4b

When approximate values are used in these problems, use the symbol ≈ in the work and answers.

1. Use the table of squares and square roots given at the end of this chapter to find the approximate value of:

   Answer:

   (a) $\sqrt{5} \approx 2.236$
   (b) $\sqrt{41} \approx 6.403$
   (c) $\sqrt{13} \approx 3.606$
   (d) $\sqrt{92} \approx 9.592$
   (e) $\sqrt{7} \approx 2.646$
   (f) $\sqrt{3} \approx 1.732$

2. Using the division method described in 7.4 find the approximate square root of the following numbers to 4 significant digits.

   (a) $\sqrt{27} \approx 5.196$
   (b) $\sqrt{77} \approx 8.775$
   (c) $\sqrt{139} \approx 11.79$
   (d) $\sqrt{68.5} \approx 8.276$
   (e) $\sqrt{9.99} \approx 3.161$

3. Using the table of square roots, find the approximate value of: (3 places)

   (a) $\sqrt{\frac{16}{4}} = \sqrt{4} = 2$
   (b) $\sqrt{\frac{25}{9}} = \frac{5}{3} \approx 1.67$
   (c) $\sqrt{\frac{100}{25}} = \frac{10}{5} = 2$
   (d) $\sqrt{\frac{3}{36}} = \sqrt{\frac{1}{12}} \approx .29$

4. Use the Pythagorean Property to find the length of the hypotenuse for each of these triangles.

   (a) Length of a is 1"; length of b is 2" .
   (b) Length of a is 4'; length of b is 5' .
   (c) Length of a is 2"; length of b is 3" .
   (d) Length of a is 5 yd, and the length of b is 6 yd.
   (e) Length of a is 1 unit and the length of b is 3 units.

4. (continued)

Answers:

(a) $c^2 = a^2 + b^2$
$c^2 = 1^2 + 2^2$
$c^2 = 5$ in.
$c = \sqrt{5} = 2.236$

(b) $c^2 = 4^2 + 5^2$
$c^2 = 41$
$c = \sqrt{41} = 6.403$ ft.

(c) $c^2 = 4 + 9$
$c^2 = 13$
$c = \sqrt{13} = 3.606$ in.

(d) $c^2 = 25 + 36$
$c^2 = 61$
$c = \sqrt{61} = 7.810$ yd.

(e) $c^2 = 1 + 9$
$c^2 = 10$
$c = \sqrt{10} = 3.162$ units

5. Sometimes the hypotenuse and one of the shorter sides is known. How can you find the length of the other side? As an example, use this problem. The hypotenuse of a right triangle is 13 ft. and one side is 5 ft. Find the length of the third side.

$c^2 = a^2 + b^2$
$13^2 = 5^2 + b^2$ Substitute values for "c" and "a."
$13^2 - (+5^2) = b^2$
$169 - 25 = b^2$
$144 = b^2$

$12 = b$. Therefore, the third side is 12 feet long. Find the third side of these right triangles. The measurements are in feet.

(a) $c = 15,\ b = 9$ (b) $c = 26,\ a = 24$ (c) $c = 39,\ b = 15$

Answers:

(a) $c^2 = a^2 + b^2$
$15^2 = a^2 + 9^2$
$15^2 - 9^2 = a^2$
$225 - 81 = a^2$
$144 = a^2$
$12 = a$

(b) $26^2 = 24^2 + b^2$
$676 = 576 + b^2$
$100 = b^2$
$10 = b$

(c) $39^2 = a^2 + 15^2$
$1521 = a^2 + 225$
$1296 = a^2$
$36 = a$

6. A telephone pole is steadied by three guy wires. Each wire is to be fastened to the pole at a point 15 ft. above the ground and anchored to the earth 8 ft. from the base of the pole. How many feet of wire are needed to stretch 3 wires from the ground to the point on the pole at which they are fastened?

Answer:

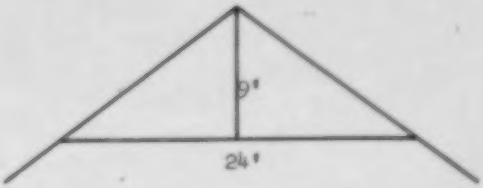
$$c^2 = a^2 + b^2$$
$$c^2 = 15^2 + 8^2$$
$$c^2 = 225 + 64$$
$$c^2 = 289$$
$$c = 17$$

3 × 17 ft. = 51 ft. total

7. A roof on a house is built as shown. How long should each rafter be if it extends 18 inches over the wall of the house?

Answer:

$$c^2 = a^2 + b^2$$
$$c^2 = 9^2 + 12^2$$
$$c^2 = 81 + 144$$
$$c = \sqrt{225}$$
$$c = 15 \text{ ft.}$$



rafter = 15 ft. + 18 in. = 16 ft. 6 in.

8. A hotel builds an addition across the street from the original building. A passageway is built between the two parts at the third-floor level. The beams that support this passage are 48 ft. above the street. A crane operator is lifting these beams into place with a crane arm that is 50 ft. long. How far down the street from a point directly under the beam should the crane cab be?

Answer:

$$c^2 = a^2 + b^2$$
$$50^2 = 48^2 + b^2$$
$$2500 - 2304 = b^2$$
$$196 = b^2$$
$$14' = b. \qquad \underline{14 \text{ feet}}$$

9. A garden gate is 4 ft. wide and 5 ft. high. How long should the brace that extends from C to D be?

Answer:

$$c^2 = a^2 + b^2$$
$$c^2 = 4^2 + 5^2$$
$$c^2 = 16 + 25$$
$$c^2 = 41$$
$$c = \sqrt{41} \approx 6.4' \text{ or } 6 \text{ ft. } 5 \text{ in.}$$

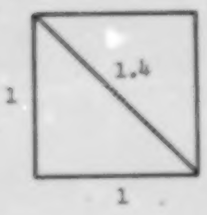
10. How long is the throw from home plate to second base in a softball game? The bases are 60 ft. apart, and a softball diamond is square in shape. Give your answer to the nearest whole foot. (Ignore the curve or arc of the ball.)

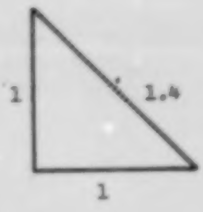
Answer:

$$c^2 = a^2 + b^2$$
$$c^2 = 60^2 + 60^2$$
$$c^2 = 3600 + 3600$$
$$c^2 = 7200$$
$$c = \sqrt{7200} \approx 84.853' \quad \text{(85' to the nearest whole foot)}$$

11. Draw a square whose sides are one unit long. Wh t is the length of the diagonal? Check by measurement. Now draw a right triangle with the sides 1 unit long. What is the length of the hypotenuse?

Answer:





7.5 Surface Area of an Equilateral Triangle

When an equilateral triangle with 2" sides is divided into two right triangles, then the line segment represented as b will be 1". (See Figure 7-5a in student text.) When angle XZY is bisected, and the bisecting line is extended through edge XY, two congruent right triangles result.

The discussion on construction of formulas is included to enable the students to arrive at surface areas of the models they have constructed.

Exercise 7-5

1. What is the area of an equilateral triangle if the measure of its edge is 3" ?

   Answer: ≈ 3.897 in.^2 ≈ 3.9 in.^2

2. If an equilateral triangle has an edge with a measure of 4.1", what would be the area?

   Answer: ≈ 7.27873 in.^2 ≈ 7.3 in.^2

3. Given an equilateral triangle with edge measure of $\frac{1}{4}$", what would be the area?

   Answer: ≈ 0.027 in.^2

4. If the area of an equilateral triangle is 10.825 in.^2, what would be the approximate measure of its edge?

   Answer: ≈ 5 in.

5. Brain buster. Given equilateral triangle ABC, with altitude CD and area of 21.2770 in.^2. Find the measure of $\overline{AB}$ and $\overline{CD}$.

   Answer: $\overline{AB}$ ≈ 7 in.
   $\overline{CD}$ ≈ 6.06 in.

6. A regular tetrahedron with an edge measure of 4 cm. has a surface area of ≈ 27.7 cm^2.

7. Find the surface area of a regular tetrahedron whose edge measure is 11". ≈ 209.6 in.^2

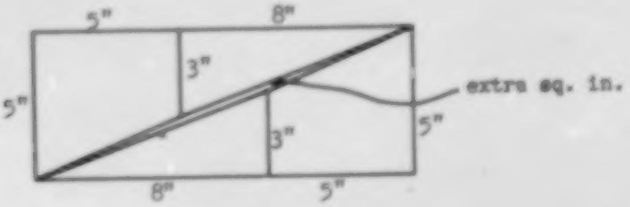
8. If the surface area of a regular tetrahedron is 339.472 in.^2 , what is the measure of its edge? ≈ 14 in.

9. Surface Area - Tetrahedron, Octahedron, and Icosahedron.

Exercise 7-51

	Number of Faces	Figure	Measurement	Area of Face	Total S.A.
Tetrahedron	4	Triangle	3 inch	3.897 in^2	15.588 in^2
Hexahedron	6	Square	3 inch	9.000 in^2	54.000 in^2
Octahedron	8	Triangle	3 inch	3.897 in^2	31.176 in^2
Dodecahedron	12	Pentagon	3 inch	15.48 in^2	185.76 in^2
Icosahedron	20	Triangle	3 inch	3.897 in^2	77.94 in^2

J.F.F. This geometric figure when cut and realigned appears to change area. It can be put together as two "triangles" 5" x 13". However, the figures are not triangles. The base is 5" , the angle is a right angle and the other side is 13", but the third side is not a straight line. It goes "in" a bit. If put together carefully, the extra square inch can be seen in the gap between the sectors.



7.6 Surface Area of a Regular Dodecahedron

The pentagonal faces of the dodecahedron are constructed with a protractor and ruler. Some student may ask about construction of a pentagon with only a compass and ruler. This is an involved construction but can be done.

The reference to trigonometry seemed necessary to develop the formula for area of a pentagon. In Sections 9-1 and 9-2 of SMSG Mathematics for Junior High School, Volume II, this concept of trigonometric ratios is presented very nicely. Rather than lengthen the section on areas, we choose to use the information and refer to later studies of trigonometry.

Exercise 7-6

1. In a regular pentagon with a side measurement of 5 in., find the area.

   ≈ 43 in.^{2}

2. If the side of a regular pentagon has a measurement of 7 in., find the area. ≈ 84 in.^{2}

3. With a length of side $3\frac{1}{2}$ in., find the area of a regular pentagon.

   ≈ 21.1 in.^{2}

4. If the perimeter of a regular pentagon is 40 cm., what would be the area? ≈ 110 cm^2

5. The perimeter of a regular pentagon is 25 in. Find the area.

   ≈ 43 in.^{2}

6. If the area of a regular pentagon is 208.12 in.^{2}, find the length of one side. ≈ 11 in.

7. Find the perimeter of a regular pentagon if the area is 497.08 cm.^{2}.

   ≈ 85 cm.

8. A dodecahedron is a regular solid with 12 pentagons as its faces. Find the surface area of a regular dodecahedron which has an edge with a measure of 6 cm. ≈ 743 cm.^{2}

7.7 Volumes of Regular Polyhedrons

The formulas for volume of regular polyhedrons required more mathematics than would be at the disposal of eighth graders. The Table 7-7 is included for them to use in finding and comparing volumes to volumes, and volumes to surface areas.

Exercise 7-7

Use Table 7-7 "Surface Area and Volume of Regular Polyhedrons" for needed information.

Example: If an octahedron had an edge measure of 2 in., its surface area would be computed by substituting into the formula for surface area of an octahedron.

$$S.A. \approx 3.464 \times s^2, \quad s = 2 \text{ in., therefore,}$$

$$S.A. \approx 3.464 \times 2^2 \approx 3.464 \approx \underline{13.86} \text{ in.}^2$$

$$Vol. \approx .471 \times 2^3 \approx 0.471 \times 8 \approx \underline{3.77} \text{ in.}^3$$

1. A tetrahedron has an edge measurement of 2 in:

(a) Find its surface area.

$$S.A. \approx 4 \times .4330 \times s^2$$
$$\approx 4 \times .4330 \times 4$$
$$\approx 6.928 \text{ in.}^2$$

(b) Find its volume.

$$Vol. \approx .118 \times s^3$$
$$\approx .118 \times 8$$
$$\approx .944 \text{ in.}^3$$

2. Find the surface area of a dodecahedron whose edge measurement is 50 mm.

Answer:

$$S.A. \approx 20.646 \times s^2$$
$$\approx 20.646 \times 50^2$$
$$\approx 51{,}615 \text{ mm}^2$$

3. What is the volume of an icosahedron which has an edge measurement of 3 cm.?

Answer:

$$Volume \approx 2.182 \times s^3$$
$$\approx 2.182 \times 3^3$$
$$\approx 2.182 \times 27$$
$$\approx 58.914 \text{ cm}^3$$

4. Find the surface area and volume of an octahedron which has an edge measurement of 5 cm.

Answer:

$$S.A. \approx 3.464 \times s^2$$
$$\approx 3.464 \times 25$$
$$\approx 86.6 \text{ cm}^2$$

$$Vol. \approx .471 \times s^3$$
$$\approx .471 \times 125$$
$$\approx 58.875 \text{ cm}^3$$

7.8 Comparison of Surface Area and Volume

The cube was returned to in this section because it

(1) can be used as a good comparison of differences in ratio between surface area and volume;

(2) the formulas for surface area and volume of a cube are not involved;

(3) the paper folding construction of cubes could be easily made without glue;

(4) in paper folding, the ratio of area of needed surface to construct a finished surface is a constant.

Table 7-8a

Volume and Surface Area of a Cube [S.A. = $6s^2$; Vol. = s^3]

Measure of the Edge (inch)	Surface Area	Volume	Ratio $\frac{\text{S.A.}}{\text{Vol.}}$	Decimal Equiv. of Ratio
12	864	1728	1 : 2	.50
9	486	729	2 : 3	.66
6	216	216	1 : 1	1.00
3	54	27	2 : 1	2.00
1	6	1	6 : 1	6.00
.5	1.5	.125	12 : 1	12.00
.1	.06	.001	60 : 1	60.00
.01	.0006	.000001	600 : 1	600.00
.001	.000006	.000000001	6000 : 1	6000.00

Table 7-8b

Name	Number of Faces	Length of Edge (in mm)	Surface Area Formula	Surface Area mm^2	Volume Formula	Volume mm^3
Tetrahedron	4	61	$1.732 \times s^2$	6444	$.118 \times s^3$	26,784
Hexahedron	6	30	$6.000 \times s^2$	5400	$1.000 \times s^3$	27,000
Octahedron	8	38	$3.464 \times s^2$	5002	$.471 \times s^3$	25,845
Dodecahedron	12	15	$20.646 \times s^2$	4645	$7.663 \times s^3$	25,862
Icosahedron	20	23	$8.660 \times s^2$	4581	$2.182 \times s^3$	26,548

Table X

Surface Areas and Volumes of Regular Polyhedrons

	Number of Faces	Shape of Faces	Surface Area	Volume
Tetrahedron	4	△	$1.7s^2$	$.1s^3$
Hexahedron	6	□	$6.00s^2$	$1.0s^3$
Octahedron	8	△	$3.4s^2$	$.8s^3$
Dodecahedron	12	⬠	$20.6s^2$	$7.7s^3$
Icosahedron	20	△	$8.7s^2$	$2.1s^3$

If we try to see a pattern from the above table, it gives us most inconclusive results. The numbers of faces increases from 4 to 6, 8, 12, 20, but if we use a unit value for s the surface area goes from 1 to 6, down to 3, up to 20 and down to 8. The volume also goes from .1 to 1, down to .8, up to 7.7 and down again to 2.1.

Because of this seeming lack of a pattern, a unit volume was chosen rather than a unit edge. Give a specific volume, the required surface area to contain it decreases as the number of faces increases. (See Table 7-21 in Teachers' Commentary.) Solids constructed with edge measures suggested in 7.21 will have an approximate constant volume and decrease in surface area as the number of faces increases.

It has been demonstrated with the calculus, that a sphere is the optimum shape to enclose a given volume of material. Soap bubbles demonstrate this fact clearly. A film of soap encloses a given volume of air. Its natural elasticity causes it to contract and shape itself into the least surface area for that volume of air it encompasses. Continuing our table to include spheres, we have

Volume of sphere $= \frac{4}{3}\pi r^3$

$1000 \approx 4.189r^3$

$239 \approx r^3$

$6.2 \approx r$

If $r \approx 6.2$, then

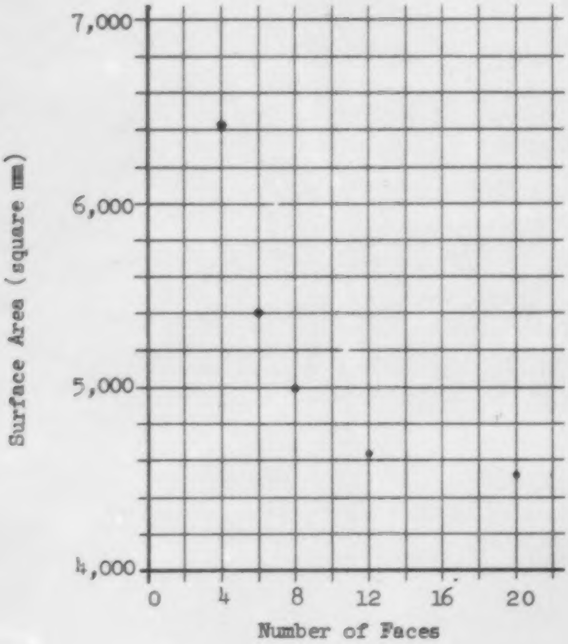
Surface Area $\approx 12.57 \cdot 6.2^2$

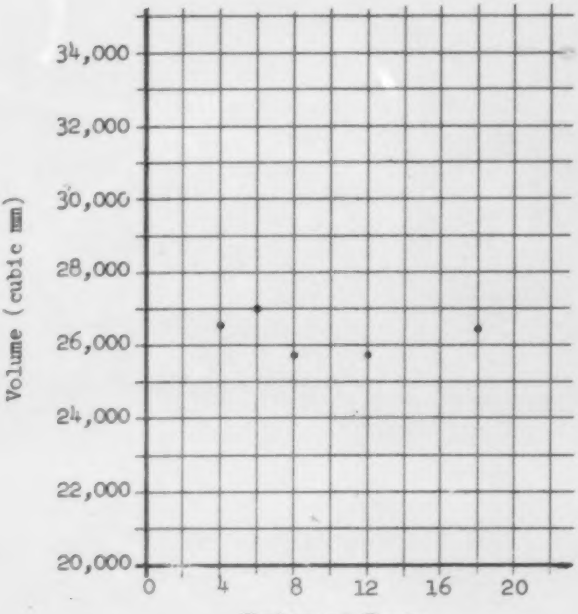
Surface Area ≈ 483 sq. cm.

Comparing the surface areas of the 5 regular polyhedrons, we find with a given volume each in turn requires less surface to enclose the volume. The converse would also be true. With a given surface area, the icosahedron would enclose a larger volume of material than the next, and so on.

Exercise 7-8

Given:





Graphs of Data from Table 7-8b, Student Text

Exercise 7-8 (continued)

Answer the following questions:

1. Do the points plotted in Graph A approximate a straight line? No
2. Which figure has the greatest Surface Area? Tetrahedron
3. Which figure has the least Surface Area? Icosahedron
4. Do the points plotted in Graph B approximate a straight line? Yes
5. What could you "generalize" from Graph B? Same capacity for all 5 solids.
6. What can you generalize from Graph A? Decreasing surface area as number of faces increases.
7. If a solid had 3 faces, would its surface area be more or less than the surface area of a tetrahedron of the same volume? More. "If" is emphasized.
8. Could a solid have 3 faces only? No. 2 faces? No.
9. As the number of faces increases, what could you say about the surface area? Decreases
10. Could a solid (not necessarily a regular solid) have more than 20 faces? Yes.
11. Could a solid have an infinite number of faces? Yes.
12. What solid would appear to approach having an infinite number of faces? Sphere
13. You have a contract for $10,000 to package 1000 cm^3 of ambergris. The packaging material costs $20 per sq. cm. What "shape" package would you like to use? (You get to keep the money you don't spend on packaging.) Sphere
14. Tough - Using the information in Problem 13, what is the most amount of money you could expect to make?

Answer: $V = \frac{4}{3}\pi r^3$ Volume = 1000 cm^3

$$\therefore 1000 = \frac{4}{3}\pi r^3$$

$$\frac{1000 \cdot 3}{4\pi^1} = r^3$$

Area of sphere $= 4\pi r^2$

$$= 4 \cdot 3.14 \cdot 6.2^2 \ (\$20)$$

$\approx \$9656$ cost of packaging with a sphere.

$1000 - $9656 = $344 max. profit

7.9 Applications to Biology

The answers to the problems where students are asked to determine the ratio between surface area and volume of cells are:

> Plant cell, a cube with a side measurement of .002 inches would have a surface area - volume ratio of 3000 : 1.
>
> Animal cell with a diameter of .008 inches would have a surface area - volume ratio of 741 : 1.
>
> Animal cell with a diameter of .016 inches would have a surface area - volume ratio of 400 : 1.

These results could lead to some very interesting class discussions on the subject of cell size in various animals and plants. It is important that the teacher realize that, in a discussion such as this, questions may arise that only a specialist in the field of cell physiology might be able to answer. If questions of this nature arise, it might be wise to inform the students that, in today's rapid increase of knowledge, no one individual could be expected to know all the answers.

If cells were to continue to grow indefinitely metabolism would become slower and slower as less and less material reached the center of the cell and less and less wastes were removed, until ultimately death would result. Again, the cells in an elephant and in a mouse are the same size, and very nearly alike. One of the astonishing facts about living things is their basic similarity. All of the cell processes (cell division, protein synthesis, use of ATP for energy, etc.) have been found to be basically alike in all living things. It is only in specialization that one finds the startling diversity among single-celled organisms as well as multicellular.

Some examples of specialization in living things are given in the following paragraph.

A Paramecium is one-celled, slipper-shaped, has cilia (tiny hair-like structures) for propelling itself through water. An Amoeba is one-celled, constantly changing shape, and propels itself by a flowing motion. An Euglena is one-celled, somewhat oval in shape, and has long thread-like flagellae for locomotion. Euglena also have chlorophyll for food manufacture, as well as a gullet for food intake. A jellyfish has tentacles for food capture. An octopus has tentacles also, but is extremely complex compared to a jellyfish. A starfish ejects its stomach into a bivalve (such as a clam or mussel), digests and absorbs the tissues in the bivalve, then withdraws its stomach. An aphid pierces the plant cells with its "beak," then sits quietly and lets the pressure within the plant pump sap into its body. A butterfly siphons nectar from a flower through a long tube. A grasshopper chews grass and other leaves. A snake swallows its food whole. A baleen whale takes a great mouthful of water, then allows it to flow out through its "strainer," the baleen, and eats the plankton it has thereby trapped. Green plants manufacture their own food, but some are tiny (like mosses), have no true roots and reproduce by spores instead of seeds. Such examples could go on and on, but the point here is that despite these numerous specializations, the cells making up these organisms are basically alike and dependent upon the same basic chemical reactions.

In answer to the question pertaining to yeast: Metabolic rates vary according to temperature in what is known as the Q_{10} ratio. As in some other chemical reactions, the rate approximately doubles as the temperature is increased 10° C. Student tests might show this tendency, but probably not to this degree of precision.

The rate of metabolism in larger animals is in general related to the surface area available for heat dissipation. Hence, the smaller the animal and the greater the surface area per unit of volume, the greater the loss of heat per unit of volume. For this reason more food must be taken in and the heart must beat faster for circulation of food and oxygen and removal of wastes.

Hummingbirds and bees are among the smaller land organisms which demand a very high rate of metabolism so need either more food or more highly concentrated food.

Sample Test Items - Chapter 7

Given:	Surface Area Formula	Volume Formula
Regular Tetrahedron	$1.732 \times s^2$	$.118 \times s^3$
Regular Hexahedron	$6.000 \times s^2$	$1.000 \times s^3$
Regular Octahedron	$3.464 \times s^2$	$.471 \times s^3$
Regular Dodecahedron	$20.646 \times s^2$	$7.663 \times s^3$
Regular Icosahedron	$8.660 \times s^2$	$2.182 \times s^3$
Sphere	$12.56 \times r^2$	$4.187 \times r^3$

Use the above table to fill in the following chart:

		Edge	Surface Area	Volume
1.	Regular Tetrahedron	20 cm.		
2.	Regular Hexahedron	10 cm.		
3.	Regular Octahedron	13 cm.		
4.	Regular Dodecahedron	5 cm.		
5.	Regular Icosahedron	7.5 cm.		

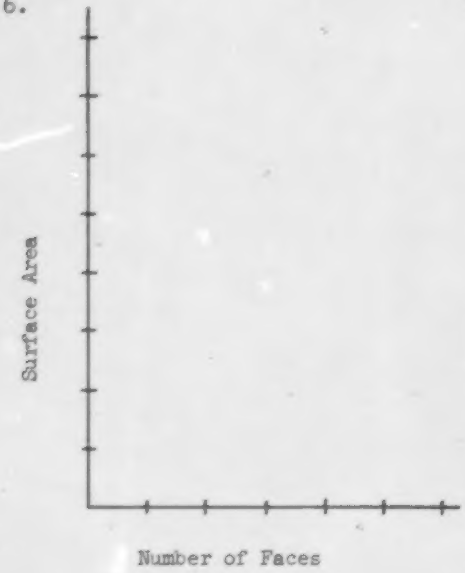
Round off to the nearest unit

Answer:

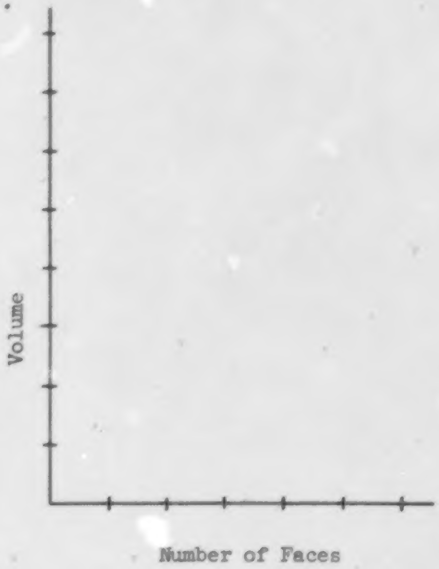
Surface A. cm^2	Volume cm^3
693	944
600	1000
585	1035
516	958
487	921

Use the above information to complete the following two graphs:

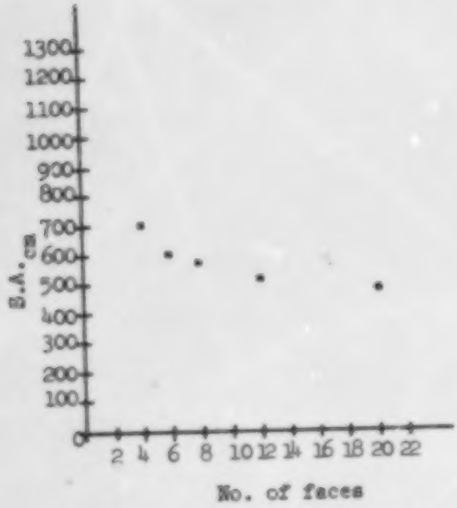
6.



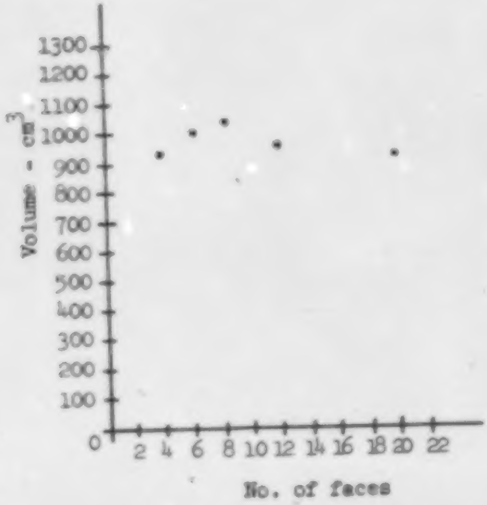
7.



6. **Answer:**



7. **Answer:**



8. What is the surface area and the volume of a sphere with a radius of 1 cm.?

(3 places) $\pi \approx 3.14$

Answer: S.A. $= 4\pi r^2 \approx 4 \cdot 3.14 \cdot 1^2 \approx$ 12.56 cm.^{2}

Vol. $= \frac{4}{3}\pi r^3 \approx \frac{4}{3} \cdot 3.14 \cdot 1^3 \approx$ 4.17 cm.^{2}

9. What is the surface area and the volume of a sphere with a radius of 4 cm.? (3 places) $\pi \approx 3.14$

Answer: S.A. $= 4\pi r^2 \approx 4 \cdot 3.14 \cdot 16 \approx 201$ cm.^{2} 201 cm.^{2}

Vol. $= \frac{4}{3}\pi r^3 \approx \frac{4}{3} \cdot 3.14 \cdot 64 \approx 268$ cm.^{3} 268 cm.^{3}

10. If a sphere has a surface area of 420 mm.^{2} and a volume of 1400 mm^3 what is the approximate ratio of surface area to volume in decimal notation (nearest tenth).

Answer: $\frac{420}{1400} = .3$ .3

Use the Pythagorean Property to find the length of the hypotenuse for each of these triangles. Find answers to nearest tenth ($a^2 + b^2 = c^2$).

11. Length of a is 4"; length of b is 6";	7.2 in.
12. Length of a is 13"; length of b is 15";	19.8 in.
13. Length of a is 60'; length of b is 60';	84.9 ft.
14. Length of a is 7"; length of b is 12";	13.9 in.
15. Length of a is 32'; length of b is 23';	39.4 ft.

Find the third side of these right triangles. Find answers to nearest tenth. ($a^2 = c^2 - b^2$)

16. c = 14 ft.; b = 7 ft.;	12.1 ft.
17. c = 128 ft.; b = 13 ft.;	127.3 ft.
18. c = 7 ft.; b = 3 ft.;	6.3 ft.
19. c = 9 ft.; b = 2 ft.;	8.8 ft.
20. c = 5 ft.; b = 4 ft.;	3 ft.

Find the area of the following equilateral triangles. (Area ≈ .4330 × s^2). Find answers to nearest tenth.

11. Length of edge (side) = 5 inches	10.8 sq.in.
12. Length of edge = 8 inches	27.7 sq.in.
13. Length of edge = 11 inches	52.4 sq.in.

Find the area of the following regular pentagons (five equal sides). Area of regular pentagon = 1.720 × s^2. Find answers to nearest tenth.

14. Side length = 4 in.	27.5 sq.in.
15. Perimeter = 50 in.	172.0 sq.in.
16. Side length = 9 in.	139.3 sq.in.

Find the area of the following dodecahedrons (12 faces, each face a regular pentagon). $(20.646 \times s^2)$ Find answers to nearest tenth.

17. Edge = 13 cm.	3489.2 $cm.^2$
18. Edge = 3 cm.	185.8 $cm.^2$
19. Edge = 7 m.	1011.7 $m.^2$
20. Edge = 2 mm.	82.6 $mm.^2$

Complete the following table:

Edge of Cube (s)	Surface Area in sq. inches	Volume in Cubic Inches	Ratio- S.A./Volume
24"	A	13,824	B : 1
15"	1350	C	D : 1
12"	864	1,728	1 : 2
9"	E	729	F : 1
G	150	125	1 :.8 1.2 : 1
H	24	I	3 : 1
J	K	1	6 : 1

Answers:

Ⓐ	3456
Ⓑ	.25
Ⓒ	3375
Ⓓ	.4
Ⓔ	486
Ⓕ	.67
Ⓖ	5
Ⓗ	2
Ⓘ	8
Ⓙ	1
Ⓚ	6
Ⓛ	1

Chapter 8

GIANT TREES:

FORMULA CONSTRUCTION FOR VOLUME OF CYLINDER AND CONE, INDIRECT MEASUREMENT

Teacher's Commentary

8.1 Introduction

The information on the big trees was obtained from two articles in the *National Geographic* and "And the Giants were Named" reprinted from the *Kaweah Magazine* and sold as a booklet in the Kings Canyon and Sequoia National Parks in California.

The General Sherman tree is considered to be the *largest* tree in existence because of its volume, not height. It is located in Giant Forest, a part of Sequoia National Park. James Wolverton named it August 7, 1879, in honor of General Sherman under whom he had served. Only about 40 per cent of the live wood still has contact with the ground. The trunk does not have the usual taper of conifers. This fact accounts for its large size. It was surveyed in 1931 and found to have a height of 272.4 ft, a diameter of 30.7 ft, and 49,660 cu ft of volume, excluding the limbs, and containing approximately 600,000 bd ft. Estimated age, found from borings, is 3500 to 4000 years.

The General Grant tree is the second largest tree in existence. It is 267.4 ft in height, 40.3 ft in base diameter, and contains 542,000 bd ft. Estimated age is 2500 years. It was named in 1867 after General Grant, who was then in command of the Union Armies. In 1926 it was formally dedicated as the "Nation's Christmas Tree."

The tallest trees are not in the Sierra Nevada Mountains of California. The tallest are called coast redwoods, *Sequoia sempervirens*, and grow in a belt barely 30 miles wide and 500 miles long on the Pacific Coast.

The world's tallest trees are in Humboldt County of northern California. The *National Geographic* magazine has a chart in the July 1964 issue:

Location	Height in Feet
Redwood Creek grove	367.8
Redwood Creek grove	367.4
Redwood Creek grove	364.3
Rockefeller tree, Redwood State Park	356.5
Founders tree, Redwood State Park	352.6
Redwood Creek grove	352.3

A few other tall trees are:

Douglas fir, Washington State	324
Eucalyptus, Tasmania	322
Eucalyptus, Australia	305

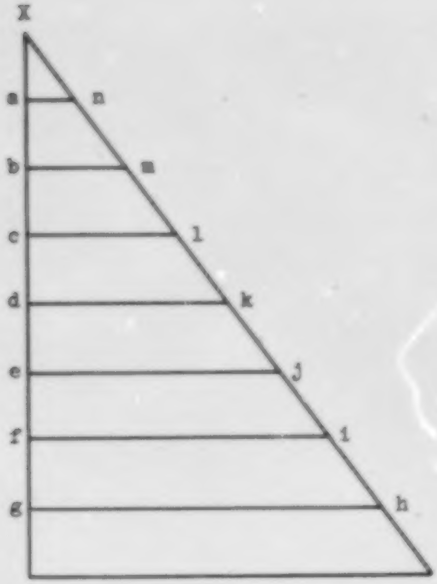
The main objectives of this entire chapter are:

1. To show the efficiency and value of formula manipulation. Devise a formula for the calculation that uses the information most easily obtained.

   "Be lazy mathematicians, not just lazy." Seek out the most efficient way to do a series of problems.
2. To develop formulas for volumes of cylinders and cones.
3. To introduce board feet as a unit.
4. To introduce indirect measurement as a tool.

8.2 Indirect Measurement

People and their shadows are used as the legs of similar triangles.



8-2a

1. What is the ratio of the base to height? $\frac{60}{80} = \frac{3}{4}$

2. If $\overline{xf}$ is 60 mm, what is the measure of $\overline{fi}$?
$\frac{3}{4} = \frac{fi}{60}$ fi = 45 mm

3. $\overline{xc}$ is 30 mm. What is $\overline{cl}$?
$22\frac{1}{2}$ mm

4. $\overline{ej}$ measures $37\frac{1}{2}$ mm. What is the measure of $\overline{xe}$? 50 mm

5. Find the measure of $\overline{an}$ if $\overline{xa}$ = 10 mm. $7\frac{1}{2}$ mm

Exercise 8-2b

1. Bob, a man 6 feet tall, has a shadow measure of three feet. A ponderosa pine tree by his cabin has a shadow measure of 124'. What is the approximate height of the pine?

   Answer: 248'

2. The "Rockefeller tree," a coast redwood tree, casts a shadow of 60 feet when a man 72 inches tall casts a shadow of 12". What is the approximate height of the tree?

   Answer: 360'

3. A young sugar pine's shadow is 9 feet. Frank, a man eight feet tall (a "big man" in that part of the country), standing beside the tree, has a shadow 12 feet long. What is the height of the tree?

   Answer: 6'

4. John, a hiker, armed with two meter sticks and a level, measured the shadow of a tall tree as 63.36 meters. He then measured the shadow of one meter stick held vertically to the earth as 72 cm. What was the height of the tree?

Answer: 88 meters

5. The shadows of Bill, Pat, and a white fir tree are in the ratio of 18, 16, 360 respectively. If Bill is 6 feet tall, how tall are Pat and the tree?

Answer: Pat is 5'4", tree is 120'

6. Choose a tree in the school yard (or flagpole if no tree). Working with a partner, measure your shadow and the tree's shadow (a) before school (b) at noon lunch hour and (c) after school. Your height and the tree's height remain the same. Only the shadow lengths vary.

Solve each separate proportion for the tree height. Find the average. Does this seem like a good way to closely approximate the height of the tree?

Answers depend on objects chosen for measurement.

8.3 Volume

This section is a general review of the units of volume based upon area of the base times the height.

In the Figure 8-3b the dimensions of the base are: length 5 m and width 4 m. The area of the base of the box would be $20\ m^2$.

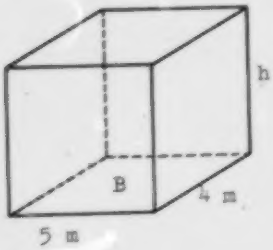


Fig. 8-3b

Filled to a height of 1 m the volume would be $20\ m^3$. 2 m height would give $40\ m^3$.

In Figure 8-3c, if the area of the base is $4\ m^2$ the volume would again be computed by finding the product of:

(area of base) × (height)

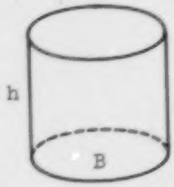


Fig. 8-3c

8.4 Area of a Circle Given the Diameter

The algebra of these substitution formulas should not be too difficult for the student to follow. He is not expected to be able to derive his own formulas but merely to follow the development in the text. The object is not to teach algebra, but rather to show how a different version of the same formula can be more useful to him.

Exercise 8-4

1. The diameter of a circle is 2 feet. Find the area.

   $\approx$ 3.14 ft^2

2. If the diameter of a circle is 14", what is the area?

   $\approx$ 153.86 in^2

3. Given the diameter of a circle as $\frac{1}{2}$ ft. Find the area in square feet.

   $\approx$ .196 ft^2

4. If 144 in^2 = 1 ft^2, give the answer to Problem 3 in square inches.

   144 × .196 $\approx$ 28.2 in^2

5. If the radius of a circle is $2\frac{1}{2}$ feet, find the area using the formula $A = .785\ d^2$.

   $\approx$ 19.6 ft^2

6. A washer has dimensions as shown in Fig. 8-4a. Find the surface area of the shaded portion. (The answer is not .785.)

   Answer: $A = \frac{\pi d}{4}$

   $$A \text{ of region} = \pi \times \frac{16}{4} - \pi \times \frac{9}{4}$$
   $$= \pi \left(\frac{16-9}{4} \right)$$
   $$= \pi\frac{7}{4} \approx \underline{5.5\ cm^2}$$

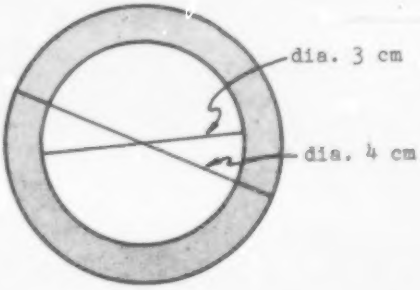


Figure 8-4a

7. In the Grant Grove of Kings Canyon National Park stands the General Grant tree with base diameter of 40.3 feet and the Texas tree, equally tall, but with base diameter of 22 feet. What is the ratio of the cross sectional areas of the General Grant tree to the Texas tree?

Answer: A of Grant tree $\approx .785 \times 40.3^2$

$\approx 1274.9 \text{ ft}^2$

A of Texas tree $\approx .785 \times 22^2$

$\approx 379.9 \text{ ft}^2$

$\frac{1274.9}{379.9} \approx 3.35$

8. Gary stepped off the distance around a giant redwood stump. He measured his step at 30" and it took 38 steps to completely circumvent the tree. What would be the area of the top of the stump?

Answer: 30 × 38 steps = 1140" circumference

$\frac{C}{\pi} = d$, therefore 1140" cir. $\approx$ 363" diameter $\approx$ 30 ft

$\underline{A \approx 706.5 \text{ ft}^2}$

8.5 Area of a Circle Given the Circumference

Recommendations for development of this section are as follows: go slowly and easily with the algebraic development; be sure each step seems reasonable to the class; do not expect these formulas to be memorized; if questions based upon the use of these formulas are included in tests, put all of the formulas on the chalkboard or on a chart.

(The common formulas, e.g., $A = \pi r^2$, $A = \frac{1}{2}bh$, etc., should be memorized by students.)

Exercise 8-5

1. The circumference of a given circle is 2 feet. Find the area.

$A \approx .08 \cdot c^2 \approx .08 \times 4 \approx \underline{.32 \text{ ft}^2}$

2. If the circumference of a circle is 8 feet, what is the area?

$\underline{5.12 \text{ ft}^2}$

3. The measure of the distance around the trunk (circumference) of the Colonel Herman cedar at Hume Lake is 23.2 ft. What is the area of such a circle?

   $\underline{43\ \text{ft}^2}$

4. Smorgasbord - (Take your choice.)

   $A = \pi r^2$ or $A = \frac{\pi d^2}{4}$ or $A = .7854d^2$ or $A = .08c^2$

   $\pi \approx 3.14$

   When A = area of a circle, r = radius, d = diameter and

   c = circumference

	Answers
a. Find A if $r = 3$ cm	$A = \pi r^2$ or $A \approx 3.14 \times 3^2$ or $\approx \underline{28.26}\ \text{cm}^2$
b. Find A if $d = 5$ mm	$A \approx .785d^2$ or $A \approx .785 \cdot 25$ or $\approx \underline{19.6}\ \text{mm}^2$
c. Find A if $c = 9$ m	$A \approx .08c^2$ or $A \approx .08 \cdot 81$ or $\approx \underline{6.48}\ \text{m}^2$
d. If $d = 6'$ find A	$A \approx .785d^2 \approx \underline{28.26}\ \text{ft}^2$
e. If $r = 7''$ find A	$A \approx 3.14r^2 \approx \underline{153.86}\ \text{in}^2$
f. If $c = 6'$ find A	$A \approx .08c^2 \approx \underline{2.88}\ \text{ft}^2$

5. Erik, a nine year old, and his 15 year old brother Andy, could just touch each other's finger tips when they reached around a sugar pine. Then they measured their combined reach. It was 10.5 feet. What would be the cross sectional area of the tree?

   $A \approx .08c^2 \approx .08 \cdot 10.5^2 \approx \underline{8.82\ \text{ft}^2}$

8.6 Volume of a Cylinder

For lumbering purposes, logs are considered to be cylinders. They are not actually circular at any one point, nor is the diameter constant. A log would more closely resemble the "frustrum of a cone" (the part of the cone bounded by the base and a plane parallel to the base).

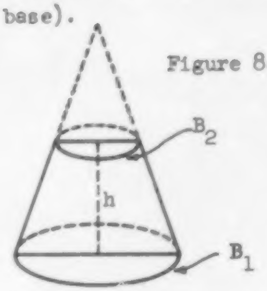


Figure 8-6

The volume of the frustrum of a cone = $\frac{1}{3}h(B_1 + B_2 + \sqrt{B_1 B_2})$. As loggers do not go into this detail, we will not either. The $\frac{1}{3}$ loss due to bark, slabs, and saw dust must be accounted for by using the diameter of the small end; loggers are assured that they do not overestimate. If a log were measured from the larger end, it would assume a board could be cut from the entire length. This would not, of course, be true.

Exercise 8-6

1. Find the volume of a log 20 ft long and 4 ft in diameter.

   Answer: $V \approx .785\ d^2h$

   $\approx .785 \times 4^2 \times 20$

   $\approx \underline{251\ \text{ft}^3}$

2. In Sequoia National Park, a home built in a fallen giant redwood tree has been named "Tharp's Log." It is not known for sure if Hale Tharp actually lived in the tree, but James Wolverton, another early day trapper and cattleman lived in the log for several years. The hollowed-out part is over 56' long and varies in height from 8' to 4'. Wolverton had it furnished with furniture, doors, and a fireplace. Before the tree fell, it was estimated to be 311 ft tall and with a base diameter of 24 ft. Assuming it was solid, what would be the volume of wood in the base 40 ft cylindrical section of Tharp's Log?

   Answer: $V \approx .785\ d^2h$

   $\approx .785 \times 24^2 \times 40$

   $\approx \underline{18086\ \text{ft}^3}$

3. Calculate the volume of wood in the bottom 24 ft log of the newly discovered "tallest tree in the world" having a base circumference of 44 feet and a height of 367.8 feet.

   $V \approx .08 \times c^2h \approx .08 \times 44^2 \times 24 \approx \underline{3717\ \text{ft}^3}$

4. Calculate the volume of wood in the bottom 24 ft log of a Michigan pine tree having a 39 inch base diameter and a height of 90 feet.

   $V \approx .785\ d^2h \approx .785 \times 3\frac{1}{4}^{\,2} \times 24 \approx \underline{199\ \text{ft}^3}$

5. What is the ratio of the volumes of problems 3 and 4 ?

   $\frac{3717}{199} \approx \underline{18.7}$

8.7 Board Feet

More discussion may be desired here about board feet. Your school's wood shop teacher would certainly urge it. It is a good topic for extensive uses of multiplication and division. "Finding of board feet" for a given piece of lumber is a good exercise. You may desire to have the students develop their own formulas for converting different sizes of lumber to bd ft. For example:

A 2" × 4" twelve inches long would contain 96 in^3 or $\frac{2}{3}$ of a bd ft.

Thus to change 18' of 2" × 4" to bd ft, multiply length by $\frac{2}{3}$. This gives 12 bd ft.

A 4" × 6" twelve inches long would contain 288 in^3 or 2 bd ft.

To change a 4" by 6" to bd ft, multiply length in ft by 2, etc.

Shop teachers often use a formula

$$\text{Board feet} = \frac{\text{length (ft)} \times \text{width (in)} \times \text{thickness (in)}}{12}$$

or

$$\text{Board feet} = \frac{\text{length (in)} \times \text{width (in)} \times \text{thickness (in)}}{144}$$

In this discussion, we omitted dimensions other than feet. Logs would not be considered otherwise except for the diameter measure. It is suggested that dimensions given in inches should be changed to fractions of a ft (decimal preferably).

Work carefully on the chalk board both ways of solving for the volume of a cylinder. One problem is impressive but each one becomes more impressive if the student is required to do it.

A better development of the entire idea would be to have in a student exercise a series of problems requiring the student to do over and over again the same computation. Thus this portion $(\pi \cdot \frac{1}{4} \cdot 12 \cdot \frac{2}{3})$ would need to be repeated in each computation. This has been reduced to 6.28 in our formula.

Exercise 8-7a

1. Find the usable bd ft in a log 18' long and with a diameter of $3\frac{1}{2}$ ft. Use both methods as described above.

Answer:

First method	Second method
$A = \pi r^2 h \times 12 \times \frac{2}{3}$	$A \approx 6.28\ d^2 h$
$A \approx 3.14 \times (\frac{7}{4})^2 \times 18 \times 12 \times \frac{2}{3}$	$A \approx 6.28 \times (\frac{7}{2})^2 \times 18$
$A \approx 3.14 \times \frac{49}{16} \times 18 \times 12 \times \frac{2}{3}$	$A \approx 6.28 \times \frac{49}{4} \times 18$
$A \approx 3.14 \times \frac{49}{2} \times 18$	$A \approx 1384.74\ \text{ft}^3$
$A \approx 1384.74\ \text{ft}^3$	

2. Mr. French has six logs on his truck. Three logs have a diameter of 42 inches each; the other three have diameters of 48 inches each. They are all 18 feet long. What is the value of the load at $55.00 per thousand usable board feet? Use the short method!

Answer:

$$\text{Bd ft} \approx 6.28\ d^2 h$$

$$\approx 6.28 \times (3\tfrac{1}{2})^2 \times 18 + 6.28 \times (4)^2 \times 18$$

$$\approx (6.28 \times 18) \left[(3\tfrac{1}{2})^2 + (4)^2\right]$$

$$\approx 113.04 \times 28\tfrac{1}{4} \times 3 \quad \text{(3 of each size log)}$$

$$\approx 9580\ \text{bd ft}$$

$$9.580 \times \$55 \text{ per m} \approx \$526.90$$

3. Find the usable board feet in the following logs:

(a) Base diameter 4 ft, length 14 ft;

(b) Base diameter 14 ft, length 22 ft;

(c) Base circumference 18 ft, length 16 ft.

Answers:

(a) $\text{Bd ft} \approx 6.28 \times 4^2 \times 14 \approx \underline{1407\ \text{bd ft}}$

(b) $\text{Bd ft} \approx 6.28 \times 14^2 \times 22 \approx \underline{27{,}079\ \text{bd ft}}$

(c) $\text{Bd ft} \approx 6.28\ d^2 h$ when $d = \frac{c}{\pi}$

$$\text{Bd ft} \approx 6.28 \left(\frac{c}{\pi}\right)^2 h$$

$$\approx 6.28 \times \frac{18^2}{\pi^2} \times 16 \approx \frac{32{,}555}{9.86} \approx \underline{3302\ \text{bd ft}}$$

Answers - Table 8-7

Length in Feet

Diameter in Feet

	8	10	12	14	16	18	20
1	50	63	75	88	100	113	126
2	201	251	301	352	402	452	502
3	452	565	678	791	904	1017	1130
4	804	1005	1206	1407	1608	1809	2010
5	1256	1570	1884	2198	2512	2826	3140
6	1809	2261	2713	3165	3617	4069	4522

Exercise 8-7b

Interpolate or extrapolate from Table 8-7 for the following problems.

1. A 16 ft log measures 52 inches in diameter. Interpolate from your table to determine the usable board feet contained in the log.

Answer:

48" 1608 bd ft $\frac{4}{12} = \frac{x}{904}$ or $x = 301$

52" ?

60" 2512 bd ft Therefore 52" dia gives 1909 bd ft

2. Compute the usable board feet for the log in Problem 1 by using your formula for the volume of a cylinder, the factor of $\frac{2}{3}$ to account for saw mill loss and compare your answer to that of Problem 1.

Answer:

$$V \approx 6.28 \times d^2 \times h \text{ bd ft}$$

$$V \approx 6.28 \times (4\tfrac{1}{3})^2 \times 16$$

$$V \approx 6.28 \times \frac{169}{9} \times 16$$

$$V \approx \underline{1887 \text{ bd ft}}$$

3. How much difference in dollar and cent value does this represent at $30.00 per thousand bd ft?

Answer:

1.909 × $30 = $57.27

1.887 × $30 = $56.61

difference = $ .66

4. Interpolate from your table to determine the lumber in an eighteen-foot log 68 inches in diameter. 3655 bd ft

5. Extrapolate from your table to determine the usable board feet in a log 16' long by 7' in diameter.

Answer:

5' dia 2512 bd ft

6' dia 3617 bd ft

7' dia ?

$\frac{1}{2} = \frac{1105}{x}$; $x = 2210$

Therefore, dia = 4722 bd ft

6. Extrapolate from your table to determine the usable board feet in a 22' log which is 5' in diameter. 3454 bd ft

7. Interpolate from your table to find the usable board feet in a 15' log $5\frac{1}{2}$ feet in diameter.

Answer:

First, for a 14' log, interpolate for $5\frac{1}{2}$ ft. Bd ft = 2682

Second, for a 16' log, interpolate for $5\frac{1}{2}$ ft. Bd ft = 3064

Third, for a 15' log, interpolate

$$\begin{array}{rr} 3064 & 2682 \\ \underline{-2682} & \underline{191} \\ 4\overline{)782} & 2873 \text{ bd ft Answer} \\ 191 & \end{array}$$

8. Check the validity of your interpolations or extrapolations in problems 4, 5, 6, and 7 by the use of the formula.

Answers:

Problem 4

$$\text{Bd ft} \approx 6.28\ d^2 h$$

$$\approx 6.28 \times (5\tfrac{2}{3}) \times 18$$

$$\approx 3630$$

Problem 5

Bd ft $\approx 6.28 \times 7^2 \times 16$

≈ 4923

Problem 6

Bd ft $\approx 6.28 \times 25 \times 22$

≈ 3454

Problem 7

Bd ft $\approx 6.28 \times (5\frac{1}{2})^2 \times 15$

$\approx 6.28 \times 30.25 \times 15$

≈ 2850

8.8 Volume of a Cone

The article on the tallest living tree is excellent for class reference. It is in the July 1964 issue of the National Geographic.

Only the trunk is considered for the volume measure. Many limbs may have a large volume, but they are not usually considered usable for lumber.

We have used the geometric shape of a cone in the approximation of volume of trees. We appreciate that a typical cone, as seen by students, has a much larger base area in relation to the height than a tree.

The final step in the development of the board foot formula for a tree (cone) is left up to the student in the text. Perhaps some students will not be able to develop their own but the intent was for all to try. If after a reasonable length of time some can not, please develop it on the chalk board for all. It is recommended that Section 8-8 be introduced near the end of a period and allow the students to develop the formula as homework.

If $V = .0265\ C^2h$	V of cone given circumference and height
$V = .0265\ C^2h \times 12$	Bd ft in a cone
$V = .0265\ C^2h \times 12 \times \frac{2}{3}$	Usable bd ft in a cone
$V = .0265 \times 12 \times \frac{2}{3}\ C^2h$	Commutative property of multiplication

[$V = .212\ C^2h$ is the formula for usable bd ft in a tree (cone)]

Exercise 8-8a

Using the formula $V \approx .0265\ C^2h$, 1 cu ft = 12 bd ft and $\frac{1}{3}$ loss of total volume at the mill, find the approximate usable bd ft in the following problems.

1. A sugar pine is 200' tall and has a base circumference of 30'.

$$\text{Bd ft} \approx .0265 \times 12 \times \frac{2}{3} \times 30^2 \times 200$$
$$\approx .212 \times 900 \times 200 \approx \underline{38{,}160 \text{ bd ft}}$$

2. A red fir has a base circumference of 14' and a height of 110'.

$$\text{Bd ft} \approx .0265 \times 12 \times \frac{2}{3} \times 14^2 \times 110$$
$$\approx .212 \times 196 \times 110 \approx \underline{4571 \text{ bd ft}}$$

3. A ponderosa pine with a height of 220' has a circumference at the base of 31'.

$$\text{Bd ft} \approx .0265 \times 12 \times \frac{2}{3} \times 31^2 \times 220$$
$$\approx .212 \times 961 \times 220 \approx \underline{44821 \text{ bd ft}}$$

4. A Jeffrey pine 120' tall has a base circumference of 10'. Find the usable bd ft.

$$\text{Bd ft} \approx .212 \times 10^2 \times 120 \approx \underline{2544 \text{ bd ft}}$$

5. Norm, a "timber cruiser" (title for the man who estimates the value of trees), found a tract of ponderosa pines: five with an average height of 200' and a circumference of 25'; seven with an average height of 100' and a circumference of 20'; fifteen with an average height of 150' and a circumference of 12'. How many usable bd ft of lumber are in the tract?

Answer:

$$\text{Bd ft} = .212\ C^2h$$
$$\approx .212 \times 25^2 \times 200 \times \underline{5} \approx 132{,}500$$
$$\approx .212 \times 20^2 \times 100 \times \underline{7} \approx 59{,}360$$
$$\approx .212 \times 12^2 \times 150 \times \underline{15} \approx \underline{68{,}688}$$

260,548 bd ft in tract.

6. If a lumber mill bids $ 20 per thousand board feet for ponderosa pine, how much would be the bid for the tract described in problem 5 ?

260.548 × 20 ≈ $ 5211

For the following problems use this information when necessary.

sugar pine	$ 30 per thousand	($ 30 per m)
ponderosa pine	$ 20 per thousand	($ 20 per m)
fir and cedar	$ 9 per thousand	($ 9 per m)

7. One thousand acres are open for bids on the timber rights. The following numbers of trees are marked for the following:

3000	sugar pines	at an average of	12,000 bd ft each
7000	ponderosa pines	at an average of	15,000 bd ft each
9000	cedar	at an average of	8,000 bd ft each
17000	fir	at an average of	4,000 bd ft each

Find the bid on the timber rights. (Would scientific notation help?)

Answer:

3000 × 12,000 × $ 30	$1,080,000
7000 × 15,000 × $ 20	$2,100,000
9000 × 8,000 × $ 9	$ 648,000
17000 × 4,000 × $ 9	$ 612,000
	$ 4,440,000

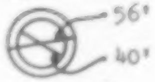
8.9 The Largest Living Things in the World

Most of the material in this section can be amplified by referring to the National Geographic magazine mentioned, although this particular article does not discuss the giant sequoias.

Information pertaining to growth rings and water transport in plants is available in the biology textbooks mentioned in the list of resource materials.

Reemphasized here is the extremely important point that scientific investigation is constantly proceeding. There are many things still unknown, areas of knowledge still unexplored, questions still to be answered. It has been noted that each time one question is answered, many more questions arise that need to be answered. This is one of the most important concepts in the world of science today and cannot be too strongly emphasized, regardless of the level at which one may be teaching. This idea of "worlds" yet to be explored can be one of the most exciting challenges a teacher can use.

Sample Test Items

Indirect measurement	Answers
1. Jim's shadow was 50" long while at the same time the shadow of his school flag pole was 50' long. Jim is 5'10". What is the height of the flag pole?	70 ft.
2. At noon on July 4 my shadow is $\frac{1}{5}$ of my height. The shadow of a radar tower at the same time (and the same place) is 63 ft. How high is the tower?	315 ft.
3. Steve, Sally, Gary, and Carol lined up for a photograph. The ratio of their shadows was 74.4, 84, 88.8, 81.6 respectively. If Steve is 62" tall, how tall are Sally, Gary, and Carol?	Sally 70" Gary 74" Carol 68"
4. Given the area of the base, the formula for the volume of a rt circular cylinder is V =	Bh
5. Given A (of a circle) $\approx .785d^2$ (d=dia) and $\pi \approx 3.14$ Find the area of a circle with dia of 14 cm	154 cm^2
6. If the radius of a circle is $3\frac{1}{2}$ feet, find the area using the formula, $A \approx .785d^2$	38.5 ft^2
7. A circular walk at Roosevelt High School has dimensions as shown. What is the area of the walk? (Use $A \approx .785d^2$.) 	1206 ft^2
Given $A \approx 3.14\ r^2$; $A \approx .785d^2$; $A \approx .08c^2$	
8. a. Find A when r = 39 mm b. Find A when d = 25 cm c. Find A when c = 42 yds.	47.76 cm^2 490.6 cm^2 141.1 yd^2
9. A large Ponderosa pine has $\approx$ 34,000 bd ft of lumber. A 75 year old Banzai Pine has $\frac{1}{119{,}000{,}000}$ the bd ft of the Ponderosa. How many bd ft in the Banzai Pine?	$\frac{1}{3}$ bd ft

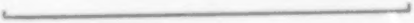
Given V of cyl $\approx 3.14\ r^2h$

$\approx .785\ d^2h$

$\approx .08\ C^2h$

10. Find the volume of a pipe which has an inside diameter of 3 ft and is 60 ft long.	$\approx 424\ ft^3$
11. The linear accelerator at Stanford is approximately 2 miles long. The inside diameter of the tube through which the charges are "shot" is 1 inch. What is the volume of the tube in cubic inches ?	$\approx 99{,}500\ in^3$
12. A heavy concrete irrigation pipe has walls 2" thick. The inside diameter is 30". Find the volume of concrete in a section of pipe 100 ft. long.	$\approx 140\ ft^3$ or $\approx 241{,}000\ in^3$
13. A corn silo has an inside circumference of 65' and a height of 30'. What is the volume?	$10{,}140\ ft^3$

SAMPLE FINAL TEST ITEMS

	Answers
1. Measure the length of this segment to the nearest cm.  (A) $5\frac{1}{2}$ cm, (B) 6 cm, (C) 67 mm, (D) none of these, (E) 7 cm.	B
2. Change 3 square feet to an equal measurement having a different unit. (A) 27 cubic feet, (B) 9 cubic feet, (C) 1728 square inches (D) 432 square inches (E) none of these	E
3. What unit of measure would be commonly used when measuring the width of a window to fit glass? (A) square feet (B) square inches (C) feet (D) $\frac{1}{4}$ inches (E) meters	D
4. If a leaf had a w/ℓ ratio of .65 and a width of 35 mm, what would be the expected length? (A) 54 mm (B) 538 mm (C) 22.75 mm (D) 227 mm (E) 185 mm	A
5. What is the greatest possible error for the following measure: 2.08 cm ? (A) ± .005 cm (B) ± .04 cm (C) ± 1.04 cm (D) ± .16 cm (E) none of these	A

6. Subtract the following:

65 mm ± 1 mm
-32 mm ± 1 mm

(A) 97 mm ± 1 mm
(B) 33 mm ± 0 mm
(C) 33 mm ± 1 mm
(D) 33 mm ± 2 mm D
(E) none of these

7. The prefix, milli- means
(A)million (B)1000 (C)100 (D)$\frac{1}{1000}$ (E)$\frac{1}{100}$ D

8. How many square centimeters are there in a square meter? (A)100,000 (B)10,000 (C)1,000
(D)100 (E)none of these B

9. How many mm^3 are there in a cc.?
(A) 10,000 (B)100,000 (C)1,000 (D)100
(E) none of these C

10. The volume of a jar is 352.8 cc. What is the mass of the water it can contain, expressed in grams?
(A)352.8 (B)35.28 (C)3.258 (D)3528 (E)none of these A

11. Write the following in scientific notation: 5687
(A)56.87 × 100 (B)5687 × 10^4 (C)5.687 × 10^3
(D)5687^2 (E)none of these C

12. Multiply. Be sure the answer has the correct number of significant digits: 487 × 5.9 (Assume these are measurements.)
(A) 2973.3 (B) 2900 (C) 2870 (D) 2873 C
(E) 3000

13. Bob's weight increased during the school year from 72 lbs to 81 lbs. What was the percent of increase?
(A)$11\frac{1}{9}$ percent (B)$12\frac{1}{2}$ percent (C)$13\frac{1}{3}$ percent
(D) $88\frac{8}{9}$ percent (E)$11\frac{1}{4}$ percent B

14. The temperature for each day one week was:
62°, 65°, 56°, 57°, 64°, 62°, and 86°. Find the D
mean temperature. (A)62° (B)65° (C)86° (D)$64\frac{6}{7}$ (E)62.5°

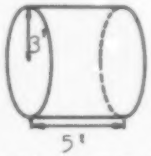
15. The temperature for each day last week was:
86°, 82°, 88°, 79°, 76°, 82°, and 85°. The mode is B
(A)82.3° (B)82° (C)88° (D)$82\frac{4}{7}$° (E) none of these

16. If the normal pulse rate was 80 and the pulse rate immediately after exercise was 100, what was the percent of increase in the pulse rate?
(A)80 percent (B)125 percent (C)25 percent C
(D)180 percent (E)none of these

17. (23,49) and (27,65) are ordered pairs. Find a second term for (26, ?). C
(A)12 (B)64 (C)61 (D)23 (E)none of these

18. Find the volume of a rectangular object that is 4 cm wide, 8 cm long, and 2 cm high.
(A)14 cm (B)64 cm^3 (C)16 cm^3 (D)34 cm^2 B
(E) none of these

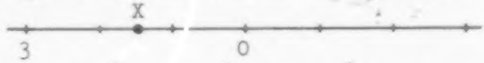
19. Find the volume of the right cylinder. The dimensions given are the radius and the height, 3 feet and 5 feet.



(A)15 sq ft (B)30 cu ft
(C)30 ft (D) 45 cu ft E
(E)none of these

20. The volume of gas in the syringe was 6 cc. Express this as cubic mm. (A).06 mm^3 (B).6 mm^3 (C)60 mm^3 E
(D)600 mm^3 (E)none of these

21. What point has been located on this line (x) ?



(A)(-1) (B)($\frac{5}{2}$) (C)($\frac{3}{2}$) (D)($-\frac{3}{2}$) (E)none of these D

22. In which quardant would you find the point represented by the following ordered pair: (7,-1)
(A) I (B) II (C) III (D) IV (E)none of these D

23. An icosahedron has how many faces?
(A)12 (B)20 (C)8 (D)4 (E)none of these B

24. The tetrahedron has a face with which shape?
(A)triangle (B)square (C)pentagon A
(D) hexagon (E)none of these

25. Use the Pythagorean property to find the hypotenuse of a right triangle if one of the legs is 9 feet long and another leg is 12 feet long.
(A)12 sq ft (B)15 ft (C)25 ft (D)21 sq ft B
(E) none of these

26. If the area of an equilateral triangle $\approx .4330 \times S^2$ then what is the area of an equilateral triangle if the measurement of its edge is 5 inches?
(A) 3.987 sq in (B)1.299 sq in (C)2.165 sq in D
(D)10.285 sq in (E)none of these

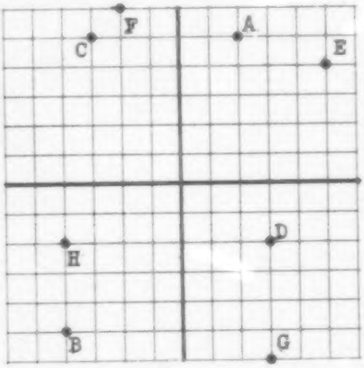
27. If the area of a regular pentagon ≈ $1.720 \times S^2$ then what is the area of a regular pentagon with a side which measures 7 inches?
(A)12.04 in (B)84 in (C)172 in^2 (D) 96 in^2 B
(E) 24.08 in

28. What solid would appear to approach having an infinite number of faces? (A)cube (B)sphere
(C)icosahedron (D)tetrahedron (E)dodecahedron B

29. If the radius of a sphere becomes twice as large the volume is (A)20 times greater (B)8 times greater (C)4 times greater (D)twice as large B
(E) stays the same

30. Plot the ordered pair represented by each point:

Point	A	(2,5)
Point	B	(-4,-5)
Point	C	(-3,5)
Point	D	(3,-2)
Point	E	(6,4)
Point	F	(-2,6)
Point	G	(3,-6)
Point	H	(-4,-2)



Resource Material

Biology

The Sierra Nevadan Wildlife Region
by Vinson Brown and Robert Livezey
Publisher: Naturegraph Co., Heraldsburg, Calif. $1.95

The Lore of Living Plants
by Johannes Van Overbeck
Publisher: Scholastic Book Services, New York $ .50

Biological Science Molecules to Man (Blue Version)
by Biological Sciences Curriculum Study (BSCS)
Publisher: Houghton Mifflin Co., Boston

High School Biology (Green Version)
by Biological Sciences Curriculum Study (BSCS)
Publisher: Rand McNally and Co., Chicago

High School Biology (Green Version)
Student Manual $1.60

Introduction to Plant Physiology
by Meyer, Anderson, Bohning
Publisher: D. Van Nostrand, Inc.

BSCS Newsletter
by BSCS
Publisher: University of Colorado, Boulder, Colo.

Plant Growth and Development
by Addison E. Lee
Publisher: American Institute of Biological Sciences,
D. C. Heath and Co., Boston $1.40

National Geographic, Vol. 126, No. 1, July 1964
National Geographic Society, Washington, D.C.

And The Giants Were Named
by Fern Gray
Publisher: Sequoia National History Association,
Three Rivers, Calif. $.25

Growth and Age, BSCS Pamphlets 16
by Louis Milne and Margery Milne
Publisher: AIBS - BSCS: D.C. Heath and Co., Boston

Van Nostrand's Scientific Encyclopedia
Publisher: D. Van Nostrand Co., Princeton, N.J.

The Green Plant
by Berner and Goodknit
Publisher: Encyclopedia Brit. Press, Chicago

Mathematics

Mathematics for Junior High, Vol. I by School Mathematics Study Group (SMSG) Yale Press	$ 3.00
Mathematics for Junior High, Vol. II by SMSG Yale Press	$ 3.00
Mathematics Through Science (Revised Edition) by SMSG A.C. Vroman, Inc., Pasadena	$ 3.00